Rapport 3/2017

Competition with Local Network Externlities

Espen R. Moen and Christian Riis, Norwegian Business School

Prosjektet har mottatt midler fra det alminnelige prisreguleringsfondet.

Competition with Local Network Externalities

Espen R. Moen and Christian Riis*
Norwegian Business School

October 31, 2017

Abstract

Local network externalities are present when the utility of buying from a firm not only depends on the number of other customers (global network externalities), but also on their identity and / or characteristics. We explore the consequences of local network externalities within a framework where two firms compete by offering differentiated products. We first show that under weak restrictions, an equilibrium exists and is unique. Second, the equilibrium allocation gives an inefficient allocation of customers on the two networks. Third, if network externalities are local, their downward pressure on prices is dampened or eliminated. Finally, local network externalities create a difference between the marginal and the average consumer, which gives rise to inefficiently high usage prices and too high levels of compatibility between the networks.

Key words: Local network externalities, differentiated products, competition, efficiency
JEL codes: D 43, D 62

1 Introduction

Network externalities are present when a user's utility from consumption of a good depends on the set of other users consuming the good. In the economics literature on network externalities, Rohlfs (1974), Katz and Shapiro (1985), Arthur (1989), Farrell and Saloner (1985, 1986), and Katz and Shapiro (1992), network externalities are primarily captured by the unidimensional variable size. In reality the composition of the network may also matter. Consumers may have preferences for the type (or identity) of the consumers in a network as well as their numbers, referred to as local network externalities. Examples of local network externalities abound.

The identity of consumers is important in traditonal network industries, such as telecommunications, when service compatibility is imperfect. Some telecommunication firms (particularly mobile

*The paper is part of a project financed by the Norwegian Competition Authority. We would like to thank Øystein Fjeldstad for invaluabale discussions and comments. We also highly appreciate comments from Michael Katz, participants at Society of Economic Dynamics annual meeting, and participants on seminars at UC Berkeley, UC Irvine, University of Oslo, and University of Toronto. Financial support from Telenor is gratefully acknowledged.

phone operators) set different on- and off-net prices. As a result, consumers prefer to subscribe to the same service as the people with whom they communicate. There is a similar effect that can be seen in the choice between platform providers. It is convenient to use the same system as colleagues and business partners. In addition, increasing returns to scale in providing applications imply that the availability of applications for a platform will depend on the preferences of its adopters, and hence consumers will tend to choose a platform where the preferences of other consumers match their own.

Other examples are found in the financial services industry, i.e. credit cards and other banking services. When choosing a credit card, the trading habits of other customers matter because they influence vendor acceptance of cards. In banking, direct and indirect transaction costs may be lower if trading partners use the same bank. In addition, a bank's customer base is a source of information that can benefit customers within the bank's area of specialization (Fjeldstad and Sasson 2010).

The examples do not stop with the traditional network industries. For consumption goods or services that involve social interaction, consumers generally have preferences regarding the identity of other customers. Obvious examples are clubs and social networking sites. For schools and universities, other customers (students) form a pool both for social interaction and a basis for a future professional network. There may be similar effects in employment decisions if the attractiveness of an employer is a function of the set of current employees.

In the present paper we analyze competition in the presence of local network externalities. In our model two firms supply horizontally differentiated products. As in the standard model, agents have preferences over product varieties, referred to as their technological preference. In addition they have preferences over the size and composition of the customer base of the firms. This is modeled by attributing to each consumer a "social location" on a circle, and letting consumers have a preference for using the same service as consumers to whom they are closely located on the circle. Finally, social location and technological preferences are assumed to be (imperfectly) correlated. An agent's choice of supplier depends on the other agents' choices, and the equilibrium is defined as a fixed point of a mapping from the other agents' choice of network to an individual's choice of network (loosely speaking). Local network effects emerge when both 1) consumers prefer to be in the same network as those who are close to them socially, and 2) the social locations and technological preferenes of individuals are correlated.

If the application at hand relates to membership in clubs, social location reflects status and foci. If it relates to the choice of platform, e.g. Apple or Windows based computers, the social location will be influenced by occupation and education. If the application at hand relates to banking, social location may reflect industry and business niche, while in mobile telephony it may be related to friends and family. Regarding computer platforms, (Apple, Windows), the technological solutions of the respective platforms may be better suited for some professional tasks than others, and thus be preferred by members of certain professions. People with whom one prefers to co-affiliate may have similar interests as oneself regarding curricula (schools), activities (clubs), and calling plans (e.g. different relative pricing of messaging and voice in mobile phone services). Hence the degree of correlation between social location and technological preferences may vary between applications.

Our paper delivers several contributions to the literature on network externalities. The first is methodological. We propose a model of competition with local network externalities. We show that under weak restrictions, equibrium exists. If the social preferences are not too strong relative to the technological preferences, we find that the equilibrium mapping is a contraction mapping, with a unique fixed point.

Our second contribution regards the welfare properties of the equilibrium allocation. We show that allocation is not socially optimal, as what we refer to as a composition inefficiency arises. Compared with the planner's solution, consumers put too much emphasis on their technological preferences and too little emphasis on their social preferences when chosing between the networks.

A third contribution regards the effects of local network externalities on competition intensity. It is a celebrated finding that network externalities may stiffen competition between firms (Gilbert 1992, Farrell and Saloner 1992, Foros and Hansen 2001, Laffont et al. 1998, Shy 2001), as network externalities increase the elasticity of the demand function. By varying the degree of correlation between technological preferences and social location, we explore how this result changes when the network externalities become more "local". We find that this tends to dampen the downward pressure of network effects on prices. The reason is that when technological preferenes and social location becomes more correlated, there are fewer marginal customers, and this weakens competition.

Finally, we show that local network externalities create a new source of divergence in the intersts of the average and marginal consumers. When technological preferences and social locations are correlated, inframarginal consumers are on average socially closer to the other consumers in the network than is a marginal consumer. As expected, this gives rise to distortions. For instance, if firms invest in enhanced one-way compatibility, firms will over-invest, because marginal agents have stronger social ties to the customers in the other network than do average customers.If firms use two-part tarrifs for connection and usage, then they will set usage prices above marginal costs. These distortions exacerbates the composition inefficiency described above.

The outline of the paper is as follows: First we give a short literature review. In section 3 we then formalize local network externalities. In section 4 we derive the aggregate demand structure with network externalities. This is not trivial, and we find it convenient first to analyze the demand facing a single platform. In section 5 we introduce competition and derive equilibrium conditions. The efficiency properties of the equilibrium allocation is derived in section 6. In section 7 we analyze how equilibrium depends on the degree of correlation between technological preferences and social locations, while we study distortions created by endogenous consumer heterogeneities in section 8. Section 9 concludes. Proof are relegated to the appendix.

2 Related literature

Some of the seminal contributors to research on network externalities were aware that network externalities need not be spillovers. Rolphs (1974) points out that there may be "communities of interest groups" where the members care mostly about the behavior of the other members in the group. Farrel and Klemperer (2007) note that "a more general formulation (of network externalities) would allow each user i to gain more from the presence of one other user j than of another k", and refers to this as local network externalities without pursuing it further. Swann (2002) assumes that different groups differ in diffusion rates and communication patterns, and on this basis shows that network effects will hardly be linear in the size of the network. A more recent related paper is Hoernig et.al. (2011) study competition with non-uniform calling patterns within telecommunication. Evans and Schmalensee (2010) allows individuals to differ regarding the intensity of their preference for the size of the network.

There exists a related literature on complex social networks, see Vega-Redondo (2007) for an overview. A much applied framework in this literature is the Watts-Strogatz (1998) model of diverse social networks. In this model, consumers are allocated on a circle. A single parameter beta indicates whether the links to other agents tend to be to the closest located agents or to agents randomly drawn from the network, i.e., how clustered the network is. This way of modelling social networks has similiarities with our model of local network externalities, as it allows for the network effects to be stronger on average the closer are the agents on the social circle. However, our model is much simpler, and gives rise to equilibria with closed-form solutions, hwile the Watts-Strogatz model requires simulations.

Lee et al (2007) use the Watts-Strogatz model to analyze whether competition between two agents eventually will lead to monopoly. By using simulations they find that monopoly is more likely in the long run the less clustered is the network. Jeho et al (2015) use the same model to study the importance of an incumbency advantage. They find that the incumbency advantage is big if the degree of clustering is low. In these two papers, the firms pricing decisions are not modelled (pricing is absent), and the analysis is therefore very different from ours.

Our paper is also related to the literature on two-sided (or multi-sided) markets, see see Rochet and Tirole (2003, 2006) and Hagiu and Spulber (2017). In fact, if each point on the social circlle is interpreted as a side, the model can be represent a market with a continuum of sides. However, we do not allow sellers to make prices contingent prices social location, which is an abstract concept (althogh some observables, like geografic location may serve as proxies). Hence our model clasifies as a network model and not a multi-sided model according to Weyl (2010). Introducing location-specific prices is on our agenda for future work.

Weyl (2010) allows for multi-dimensional heterogeneity among agents, and the attractiveness of the platform depends on customers characteristics as well as the participation from the other sides of the market. Our model differs from Weyl's in several ways. Most importantly, we do not allow prices to be "insulating", meaning that they are contingent on the number of agents of other types that enter the market. This is crucial for our welfare result. Second, we explicitely focus on the relationship between social location and social closeness between agents, and how social locations and and technological preferences are correlated. Similar results are not found in Weyl's paper. Finally, we allow for a continuum of social locations, and our proof of existence of equilibrium thus constitutes a methodological contribution absent in Weyl.

In the literature on two-sided markets, it is known that cross-over effects between the sides may influence prices. Chandra and A. Collard-Wexler (2009) shows that mergers of multisided firms may reduce prices. Armstrong (2006) show that if the agents on side 2 of the market obtains utility from having more agents entering from side on of the market, this will cet par lead to lower prices on side 1 of the market. In our paper, the platform cannot discriminate between the different customers. Furthermore, none of these papers have analyzed the effects of the degree of correlation between technological preferences and social location, i.e., the degree of "localness" influences incentives to undertake compatibility-enhancing investmens as well as the optimal price structure. More local network effects lead to fewer marginal customers, this effect is absent in Armstrong (2006) and the other papers we know of.

It is known from the literature that differences in the characeristics of marginal and average differences may distort the allocation of resources. This was first demonstrated by Spence (1974) with several follow-ups in different economic settings. Hoering et. al. (2011) study distortions created by calling circles. Weyl (2011) study how marginal customers may differ from average customers with two-sided hetrogeneity, in which agents differ both in their overall willingness to pay to enter a market as such and their weight on size of the customer pool on the other side. This two-sided hetrogeneity implies that the marginal customers are less concerned with size effects than the average customer, and this leads to distortions in the pricing decisions. In our model, by contrast, all agents have the same intensity of social preferences or size (although they differn regarding their preferences for the identitiy of the other customers). However, the degree of "localness" influences to what extent the custonmers care for compatibility with the agents in the other network.

There is ample empirical evidence that local network externalities are important. Birke and Swann (2005) study individual consumers' choice of mobile operators in the U.K. They find that individual choices are heavily influenced by the choices of others in the same household. Tucker (2008) analyzes the introduction of video-messaging technology in an investment bank. She finds that adoption by either managers or workers in boundary spanner positions has a large impact on the adoption decisions of employees who wish to communicate with them. Adoption by ordinary workers has a negligible impact. Corrocher and Zirulia (2009) survey Italian students' choice of mobile operator and find that local network effects (the choice made by friends and family members) play an important role, although the strength of the effects is heterogeneous.

3 Modelling local network externalities

Althoug our aim is to study competition, it is convenient to start by studying the demand facing a single network / platform (the terms network, platform and firm will be used interchangeably) that offers a set of consumers /customers/agents access to a network (the terms will be used interchaneably). The extension to oligopoly then follows straightforwardly.

The value of being connected to the network depends on an agent's intrinsic valuation of the network services, as well as the number and the identity of the other agents that connect to the network. We refer to the first as a consumer's technological preferences, and the second to the customers social preferences. The consumers /consumers /agents (we use the terms interchangeably) are hetrogenous along both dimensions. The consumers' intrinsic preferences for the network are represented by the parameter y, which is continuously distributed over the consumers. The parameter y represents the "travel cost" of the agent. A higher value of y thus indicates that everything else equal, the agent likes the network less.

An innovation in this paper is that we introduce social preferences. Consumers' social preferences are represented by a Salop circle, with circumference equal to one.[1] Each consumer has a *social location* (or just location) on this circle. When specifying the agents' location on the circle, we find it convenient to specify whether the agent is on the east or the west hemisphere (although in the proofs in the appendix we use different notations). To be more precise, denote agent i's social location by z_i^k, where $k = E, W$ indicates the *east* and *west* hemisphere. We denote by Ω the full circle, and Ω^k the k-hemisphere, where k is either east or west. We refer to $-1/4$ as the south pole and $1/4$ as the north pole.

[1] The motivation behind letting agents be distributed on the circle is to avoid the asymmetry associated with consumers on the end of a line that only communicate in one direction.

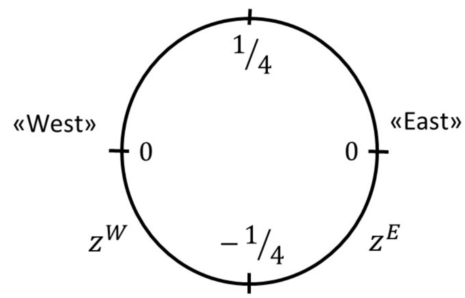

Figure 1

Let d denote the distance between two locations on Ω - referred to as social distance. The distance between two agents located on the same hemisphere is

$$d(z_i^k, z_j^k) = |z_i^k - z_j^k|$$

The distance between two agents located on opposite hemispheres (k and k^c respectively) is

$$d(z_i^k, z_j^{k^c}) = \min\left[\frac{1}{2} - z_i^k - z_j^{k^c}, \frac{1}{2} + z_i^k + z_j^{k^c}\right]$$

By symmetry, the western hemisphere is a perfect mirror image of the eastern. Therefore, for notational convenience we suppress the topscript k when this does not lead to confusion.

A driving assumption in our analysis is that social and technological preferences may be related. We assume that people who are socially close are more likely to share the same technological preferences. Let $F^z(y) \equiv F(y|z)$ with support given by $[y^{\min}(z), y^{\max}(z)]$ denote the conditional distribution of y among the buyers with social location z (note that the distribution is the same at the eastern and the western hemisphere).

We assume that $F^z(y)$ is continuously distributed with density $f^z(y)$, and that there exists an upper bound $f^{\max}$ such that $f^z(y) \leq f^{\max}$ for all z, y. We also assume that $\partial F^z/\partial z$ is well defined, and that $|\partial F^z(y)/\partial z|$ has an upper bound denoted by f^z, i.e., that $|\partial F^z(y)/\partial z| \leq f^z$ for all z, y.

Let the continous function $g : [0, 1] \rightarrow R^+$ denote agent i's utility of having an agent at social distance d in the network. Unless stated otherewise we assume that g is strictly decreasing in d, reflecting that agents gain more from "being together" with people that are socially close than socially distant. Hence the network externalities are local in the sense that they are stronger between the closer the agents are located in social space.

A negative g is not allowed. Hence there are no crowding-out effects of membership. This seems to be a reasonable assumption for platforms, banks, and telephony, but may be less so for social clubs, where the average member "type" may matter. Note also that this additivity property gives rise to increasing returns to scale on the demand side. Suppose a fraction $\tilde{H}(z)$ of the agents of social location z belongs to the network (or, alternatively, the probability that a person located at z chooses the network).[2] Below we refer, somewhat impreciciely, to $\tilde{H}$ as the distribution of customers on the network, or just the distribution function. The social utility of joining the network for this person is

$$\int_{\Omega} g(d(z, z_i))\tilde{H}(z)dz.$$

For notational simplicity, the subscript Ω is dropped in all integrals from now on. Finally, define $\mathbf{g}$ as

$$\mathbf{g} \equiv \int g(d(z, z_i))dz \tag{1}$$

where $\mathbf{g}$ denotes the social utility a consumer obtains if all agents in the economy join the network. If $g(d) = g$ for all d, we say that the network effects are *global*. We say that g_2 is more concentrated than than g_1 if g_1 can be constructed by performing a mean-perserving spread of g_2, i.e, by decreasing $g_2(d)$ for low values of d and increasing $g_2(d)$ for high values of d in such a way that $\mathbf{g}$ stays constant. If g_2 is more concentrated than than g_1 we say that g_2 represents more *local network externalities* than g_1, or simply that the network externlites are more local. We also consider the limit, in which $g(0)$ goes to infinity and $g(d)$ goes to zero for all $d > 0$, keeping $\mathbf{g}$ constant. We refer to this limit as a situation with *pure local network externalities.*

Occasionally we refer to $\mathbf{g}$ as an individual's total number (measure) of "friends". The value of being in the same network as a friend is then normalized to 1. With this interpretation, may be interpreted as the probability density that a person has a friend (or the number of friends) at distance d.

[2] At this point our model allows for two different interpretations. Either there may be one person located at each z, in which case $H(z)$ is a probability. Or it may be a continuum of agents with measure 1 at each z, in which case $H(z)$ is a fraction. We will use the two interpretations interchangeably.

4 Demand

Suppose the platform sets a price p of entering the network, and that the agents independently decide whether to join the network, given the prices and given rational expectations about the choice of the other agents in the economy.[3] The utility of an agent with characteristics (y_i, z_i) joining the network at price p, is given by

$$u(y_i, z_i) = \beta + \int g(d(z, z_i))H(z)dz - y_i - p \tag{2}$$

where β is a given strictly positive parameter. The payoff obtained by not joining the network is normalized to zero.

Let $H_0(z)$, $0 \leq H_0(z) \leq 1$ be an arbitrary continuous distribution function. Let $y^m(z_i; H_0)$ denote the technological preference of the indifferent agent at location z_i. If all agents at z_i prefer to join the network, let $y^m(z_i; H_0) = y^{\max}(z_i)$. If no agent prefers to join the network, let $y^m(z_i; H_0) = y^{\min}(z_i)$. Otherwise, $y^m(z_i : H_0)$ is defined by the equation $u(y^m(z_i; H_0), z_i) = 0$, i.e., by the equation

$$y^m(z_i; H_0) = \beta + \int g(d(z, z_i))H_0(z)dz - p. \tag{3}$$

Define $H_1(z_i)$ as the fraction of agents at social location z_i that joins the platform, as a function of the distribution function H_0, we write $H_1(z_i) = \Gamma H_0$. By definition, $H_1(z_i) = F(y^m(z_i; H_0))$. If $y^m(z_i; H_0) = y^{\min}(z_i)$, then $H_1(z_i) = \Gamma H_0(z_i) = 0$. If $y^m(z_i; H_0) = y^{\max}(z_i)$, then $H_1(z_i) = \Gamma H_0(z_i) = 1$. Otherwise,[4]

$$\Gamma H_0(z_i) = F^z \left(\int g(d(z, z_i))H_0(z)dz + \beta - p \right). \tag{4}$$

Since the integral of a continuous function is continuous, it follows that $\Gamma H_0(z_i)$ is continuous in z_i. For a given price p the *equilibrium distribution function* $H(z)$ is a fixed-point satisfying

$$H = \Gamma H.$$

In the appendix we show that Γ is equicontinuous on its domain, and hence that Schauder's fixed-point theorem applies:

[3] For a discussion of expectations formation in markets with network externalities, see Griva and Vettas (2011).

[4] Note that since $F^z(y) = 0$ for all $y \leq y^{\min}$, and $F^z(y) = 1$ for all $y \geq y^{\max}$, (4) defines Γ for all H_0 and all z_i.

Proposition 1 *For any given price p, Γ has a fixed point, hence an equilibrium distribution function exists.*

Proposition 1 ensures existence of an equilibrium distribution, but not uniqueness. In order to show uniqueness, we have to impose further parameter restrictions. More specifically, we require that $\mathbf{g} < 1/f^{\max}$. In the appendix we show that Γ is a contraction under the sup norm on a complete matrix space. Hence it follows from the Banach fixed point theorem (the contraction mapping theorem) that Γ has a *unique* fixed point.[5]

Proposition 2 *Suppose $\mathbf{g} < 1/f^{\max}$. Then for any given price p, the fixed point $H = \Gamma H$ exists and is unique.*

Proof. See appendix ■

To gain intuition, suppose as an example that F^z is uniform with density $f < 1/\mathbf{g}$, and that all types increase their threshold value $y^m(z)$ with Δ units (from now on we supress the dependence of y^m on H_0). This increases H with $f\Delta$ units. The increased utility of joining network H due to network externalities is thus $f\mathbf{g}\Delta$. The increased cost for the marginal agent however is Δ, which is greater than $f\mathbf{g}\Delta$ by assumption. Hence demand is stable in the flollowing sense: an increase in the number of agents that choose to connect to the network increases the attractiveness of the network, but not sufficiently much to compensate for the loss associated with the increased value of y for the new agents.

Let Γ^n denote the mapping that emerges when Γ is applied n times. Since Γ is a contraction, we know that for any distribution function H_0, the fixed-point H is uniquely defined as $H = \lim_{n\to\infty} \Gamma^n H_0$.

5 Competition

Suppose now that there are two platforms A and B that offer services, so that the alternative may be to join the other platform. Furthermore, we assume that the market is covered. Hence at any z, we let $H(z)$ denote the fraction of the customers who join network A, and $1 - H(z)$ the fraction

[5] Above we have defined Γ on the set of continuous functions. One may ask whether Γ may have another fixed-point $\widetilde{H}$ that is not a continuous function. However, for any integrable function $\widetilde{H}$, $\Gamma\widetilde{H}$ is continuous. Hence Γ does not have a non-continuos (integrable) fixed point.

that joins network B. We assume that the social value of joining the two platforms have the same structure, and equal to $\mathbf{g}_A(z_i) = \int g(d(z, z_i))H(z)dz$ and $\mathbf{g}_B(z_i) = \int g(d(z, z_i))\,(1 - H(z))\,dz$, respectively. Since the market is covered it follows that $\mathbf{g}_A(z_i) + \mathbf{g}_B(z_i) = \mathbf{g}$. We redefine y to be an agent's technological preference for the B-platform over the A-platform. An agent located at z_i obtains utilities of joining network A and network B equal to $u_A = \beta + \mathbf{g}_A(z_i) - y - p_A$ and $u_B = \beta + \mathbf{g}_B(z_i) - p_B$, respectively, and choses network A whenever

$$\begin{aligned} y &\leq \mathbf{g}_A(z_i) - \mathbf{g}_B(z_i) + p_B - p_A \\ &= 2\mathbf{g}_A(z_i) - \mathbf{g} + p_B - p_A. \end{aligned}$$

since $\mathbf{g}_A(z_i) + \mathbf{g}_B(z_i) = \mathbf{g}$.

Let $y^m(z_i)$ denote the technological preference of the indifferent customer with social location z_i. It follows that $y^m(z_i) = \mathbf{g}_A(z_i) - \mathbf{g}_B(z_i) + p_B - p_A$ (if all /no agents in the support prefer the A network, then $y^m(z_i)$ is equal to $y^{\max}(z_i)$ or $y^{\min}(z_i)$, respectively). As above, the distribution H is given as fix-point to a mapping of the form $H_1(z) = \Gamma H_0(z)$. Since $H(z_i) = F^z(y^m(z_i))$, we have that H is the solution to the fix-point

$$\Gamma H(z_i) = F^z(2\int g(d(z, z_i))H(z)dz - \mathbf{g} + p_B - p_A). \tag{5}$$

The mapping is analogous to the equilibrium mapping (4). The factor 2 reflects that the alternative now is the value associated with joining the B-network, where the complementary part of the customers are located. The formal structure of the mapping is the same as above, with $\beta - p$ replaced by $-\mathbf{g} + p_B - p_A$ and g by $2g$. Hence it follows that the mapping is a contraction if $2f^{\max}\mathbf{g} < 1$.

Corollary 1 *Suppose* $2f^{\max}\mathbf{g} < 1$. *Then the equilibrium distribution* $H(z_i)$ *defined by (5) has a unique solution.*

Let N_A and N_B denote the total number of agents in network A and B, respectively. Then

$$\begin{aligned} N_A(p_B - p_A) &= \int H(z; p_B - p_A)dz \\ N_B(p_B - p_A) &= \int [1 - H(z; p_B - p_A)]\,dz = 1 - N_A(p_B - p_A) \end{aligned}$$

In the appendix we show that $N_i(p_B - p_A)$ is continuous in $p_B - p_A$. The profit of firm A and B can be written

$$\begin{aligned} \pi_A &= (p_A - c) N_A(p_B - p_A) \\ \pi_B &= (p_B - c)\left[1 - N_A(p_B - p_A)\right] \end{aligned}$$

with first order conditions for maximum given by

$$N_A(p_B - p_A) - (p_A - c) N_A'(p_B - p_A) = 0 \tag{6}$$

$$1 - N_A(p_B - p_A) - (p_B - c) N_A'(p_B - p_A) = 0 \tag{7}$$

Proposition 3 *With identical costs, in a pure strategy equilibrium, the solution is uniquely determined by the two equations given by*

$$p_A = p_B = c + \frac{1}{2N_A'(0)}. \tag{8}$$

The second order condition for firm A reads

$$-2(p_B - p_A) N_A'(p_B - p_A) + (p_A - c) N_A''(p_B - p_A) < 0 \tag{9}$$

The second order condition for firm B is defined analogously. Due to symmetry, $N_A(\cdot)$ is odd, and thus has an inflection point at zero. Hence $N_A''(0) = 0$, and the second order conditions are satisfied locally.

To show that the equilibrium is the unique equilibrium in pure strategies, from (6) and (7) it follows that $\frac{N_A}{N_B} = \frac{p_A - c}{p_B - c}$. As the right hand side is incrasing in p_A, and the left hand side is decreasing in p_A, it follows that the symmetric solution is the unique pure strategy equilibrium.

As an example, suppose $F^z(y) = F(y - az)$, and that F is uniform with density f. Hence the support of $y(z)$ is given by $[az - 1/f, az + 1/f]$. If $0 < H(z) < 1$ for all z, we have that

$$N_A'(\cdot) = \frac{f}{1 - 2f\mathbf{g}}$$

Since $N_A'(\cdot)$ is a fixed number, $N_A''(\cdot) = 0$, and the second order condition for the pure strategy equilibrium holds globally.[6] Hence the equilibrium price is

$$p = c + \frac{1}{2f} - \mathbf{g}. \tag{10}$$

[6]It is straight forward to show that $N_A''(\cdot) \leq 0$ if, in equilibrium, $H(z)$ equals 1 or 0 for some z.

When the demand functions are sufficiently non-linear, it is well known that competition between firms with differentiated products may not have pure strategy equilibria, see e.g. Laffont and Tirole (1998). In what follows we assume that parameters are such that equilibrium in pure strategies exists.

6 Equilibrium distributions

In this secition we will derive some properties of the equilibrium distribution H and the equilibrium mapping Γ at $\Delta p = 0$. In addition to being interesting in its own right, the shape of H has concequences for equilibrium prices, to be discussed below.

In what follows we make the following assumptions on $F^z(y)$:

1. Higher z is associated with higher values of y: If $z_1 > z_0$, then F^{z_1} first order stochastically dominates F^{z_0}.

2. Symmetry around equator ($z = 0$): $y|z$ and $-y|-z$ are identically distributed.

3. In some applications we also assume that $F^z(y) = F(y - az)$ as indicated above. The density $f(y - az)$ is assumed to be single-peaked at and symmetric around $y - az = 0$

   When requirement 3 is imposed, we will, somewhat imprecicly, refer to a as the degree of correlation between social location and technological preferences. If $a = 0$, the two are independent.

Let D denote the set of distribution functions $H : [-1/4, 1/4] \to [0, 1]$ satisfying 1)-3). We say that a distribution function $H^1 \in D$ is *more concentrated* than a distribution function $H^0 \in D$ if $H^1(z_i) \geq H^0(z_i)$ for all $z_i < 0$, with strict inequality if $H^0(z_i) < 1$, and $H^1(z_i) \leq H^0(z_i)$ for all $z_i > 0$, with strict inequality if $H^0(z_i) > 0$.

We know *ex ante* that the equilibrium distribution functions are 1) continuous, 2) antisymmetric around $(z = 0, H = 1/2)$, i.e., such that $H(z_i)+H(-z_i) = 1$. The determinants of the concentration of $H(z)$ is key for our analysis, and for this purpose the following lemma is convenient:

Lemma 1 *Suppose $H^0 \in D$. Let $H^1 = \Gamma H^0$. Then $H^1 \in D$. Furthermore, suppose H^1 is more concentrated than H^0. Then $H^2 = \Gamma H^1$ is more concentrated than H^1.*

From the definition of Γ and symmetry, $H^1 = \Gamma H_0$ satisfies requirement 1) and 2). The rest of the lemma is proved in the appendix.

The lemma is very helpful, as it implies that when analysing the effects of a shift in a parameter on the concentration of the equilibrium distribution, it is sufficient to study the "first-round effect" of a shift. To be more specific, suppose Γ_1 and Γ_2 are two mappings, and H_1 and H_2 the corresponding equilibrium distributions. Recall that $H_2 = \lim_{n\to\infty} \Gamma_2^n H_0$. Then if follows from the lemma that if $\Gamma_2 H_1$ is more consentrated than H_1, then H_2 is more concentrated than H_1.

Suppose for instance that $H^0(z_i) \equiv 1/2$. Then $H^1 = \Gamma(H^0) = F^z(\int 2g(d(z, z_i)dz - \mathbf{g})$ $=F^z(0)$. Due to the stochasatic dominance assumption, it follows directly that H^1 is more concentrated that H^0 (if $F^z(y) = F(y - az)$, it follows that $H^1(z) = F(-az)$. The first part of the next corrolary follows directly, the second part is proved in the appendix.

Lemma 2 *Suppose requirement 1-2 are satisfied. The the equilibrium distribution $H(z)$ is decreasing in z, and strictly decreasing whenever $H \in (0, 1)$. If requirement 3 is also satisfied,then $H(z)$ is concave for $z < 0$ and convex for $z > 0$.*

If social preferences and technological preferences are independent, then in equilibrium $H(z_i)$ is constant for all z_i. Any agent has a fixed proportion of her friends in the network. If social and technological preferences are correlated, the pattern in the figure appears. The equilibrium function $H(z)$ satisfies requirements 1)-3) stated above.

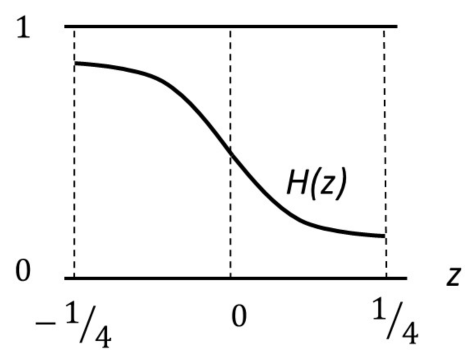

Figure 2

Due to technological preferences, agents close to the south pole are more likely to join the A network. The social preferences reinforce this and creates a multiplier effect: when more agents enter the A network, it is more attractive to enter the network for other customers, and particularly so for agents that have a close social location. This effect increases as z_i moves towards $-1/4$ (closer to south pole), but at a decreasing rate (given that requirement 3 is satisfied). The existence of social preferences increases H and gives it a concave shape at the southern hemisphare, while it decreases it and gives it a convex shape at the northern hemsphare.

A main concern is how the concentration of H depends on the network effects, both their overall importance (measured by $\mathbf{g}$) and the extent to which they are local (the concentration of g). We first analyze the effects of an increase in overall importance. To this end, define $g(z) \equiv k\bar{g}(z)$, where k is a shifter. We say that networ effects become more important if k increases. Then the follwing holds

Lemma 3 *a) An increase in k, the importance of network effects, makes $H(z)$ more concentrated.*

The proof is given in the appendix. An informal description of the proof goes as follows: Consider two values of k, k^h and k^l. If we plug the equilibrium distribution for $k = k^l$ into the equilibrium mapping for $k = k^h$, it follows that more customers will join the network for $z < 0$ and fewer for $z > 0$, since network effects have been more important relative to technological preferences. Hence the new distribution is more concentrated than the equilibrium distribution for $k = k^l$. Now we can apply the equilibrium mapping for $k = k^h$ repeatedly, and in the limit obtain the equilibrium distribution for k^h, and lemma 3 ensures that the first-round effect "survives" and that the equilibrium distribution for $k = k^h$ is more concentrated than the equilibrium distribution function for $k = k^l$.

To gain (more) intution, note that customers below $z = 0$ have a tendency to prefer the A network for technical reasons (low y), while the opposite is true for $z > 0$. This is reinforced by the network externalities, the customers below $z = 0$ prefer to be in the same network as other customers at $z < 0$, while the opposite holds for $z > 0$. An increase in k increases these network effects, and locates even more $z < 0$-customers and even less $z > 0$- customers to A.

The next important observation regards the role of the correlation between social and technical preferences, captured by the parameter a. The following holds:

Lemma 4 *If $a = 0$, $H(z) = 1/2$ for all z. An increase in a makes $H(z)$ more concentrated.*

An increase in a strengthens the tendency for customers at $z < 0$ to perfer the A network and for customers at $z > 0$ to prefer the competitior. The "first-round" effect of an increased a is therefore to shift H up (down) for z below (above) 0. The first round effect is then fortified by the multiplier effect caused by the network externalities. In the extreme case with $a = 0$, location does not influence the choice of platform, and hence there is no initial distributional effect that can be reinforced by network effects. Hence $H(z)$ is flat and equal to $1/2$ for all z.

Finally we consider how the concentration of g influences the concentration of H. In the appendix we show the following:

Lemma 5 *Suppose assumptions 1)-3) are satisfied. Suppose g_2 is more concentrated than g_1. Let H_1 and H_2 represent the associated equilibrium distribution functions. Then H_2 is steeper than H_1.*

This result follows from our finding that $H(z)$ is concave, see lemma 2. The result is rather intuitive at the poles. Recall that $H(z)$ is high around and highest at $z = -1/4$, and is decreasing as z increases. Hence the more concentrated g is, the higher is the fraction of friends a person at $z = -1/4$ has in the A-network, and the more attractiv it is for this person to join the A-network. The same effects are at play for any $z < 0$, while the opposite is true for $z > 0$, here it is the B-network that is advantaged by a more concentrated g.

It follows that the most concentrated H-distribution is obtained in the limit when the network effects are pure local. In this case, all the friends of the person are located at the same social location as himself. It follows that $\int H(z)g(d(z, z_i)dz = \mathbf{g}H(z_i)$. It follows from (5) that

$$y^m(z_i) = \mathbf{g}[2F(y^m - az_i) - 1]$$

This equation is not particularly easy to solve, except in the not so interesting case in which F is uniform.[7]

[7] Suppose F is uniform, and given by $F(y) = \frac{1}{2} + (y - az) \times f$, where f is a constant. Provided that y^m is in the support of F, it follows that

$$y^m(z_i) = -\frac{2\mathbf{g}az_i}{1 - 2\mathbf{g}fz_i}$$

7 Efficiency

In this section we analyze the efficiency properties of equilibrium. First we derive the optimal distribution of agents over networks, and refer to this as composition efficiency. At any given social location z_i, a fraction $H(z_i)$ of the agents join network A, hence the *total (gross) social value* created in network A, V_A, is

$$V_A = \int \mathbf{g}_A(z_i)H(z_i)dz_i$$

Analogously, denote the total social value created in network B by V_B . Then

$$V_B = \int \mathbf{g}_B(z_i)(1 - H(z_i))dz_i$$

In the appendix we characterize the allocations of agents on networks that give the highest and the lowest total social value, given that the two networks are equally large. The total social value is *minimized* if $H(z) = 0.5$ for all z, in which case each agent can communicate with exactly half of her friends. The social value is *maximized* if $H(z)$ equals 1 on an interval with measure 1/2, and is zero on the complementary interval. However, the allocation that maximizes total social value implies that some of the agents are allocated to a network with a technology they disfavor. Hence there is a trade-off between the social benefits of increasing the number of connections and costs associated with not allocating consumers according to technological preferences.

For a given distribution $H(z)$, let $Y(z)$ denote aggregate technological utility for agents located at z,

$$Y(z) = -\int_{-\infty}^{y^m(z)} yf^z(y)dy. \tag{11}$$

Finally, define $\mathbf{Y} = \int Y(z)dz$.

A *composition efficient* distribution, denoted by $H^*(z)$, maximizes social welfare defined as

$$\begin{aligned} W &= V_A + V_B + \mathbf{Y} \\ &= \int [\mathbf{g}_A(z)H(z) + \mathbf{g}_B(z)(1 - H(z)) + Y(z)]dz \end{aligned} \tag{12}$$

In the appendix we show that point-wise maximization gives the following first-order condition:

$$H^*(z_i) = F^z\left(2\left[2\int g(d(z,z_i))H^*(z)dz - \mathbf{g}\right]\right)$$

Thus $H^*(z)$ is a fixed-point to the mapping Γ^g given by

$$\Gamma^g H^*(z_i) = F^z\left(2\left[2\int g(d(z,z_i))H^*(z)dz - \mathbf{g}\right]\right) \tag{13}$$

The planner's fixed-point is identical with the fixed-point that determines the market solution, with the exception that the weight on network effects is doubled. From lemma 3 part 1), the next proposition therefore follows directly

Proposition 4 *The equilibrium distribution is not composition efficient, as the socially optimal distribution is more concentrated than the equilibrium distribution.*

The efficiency result is intuitive. Recall that the equilibrium distribution when $p_A = p_B$ is the fix-point to the mapping (from (4)

$$\Gamma H(z_i) = F^z\left(2\int g(d(z,z_i))H_0(z)dz - \mathbf{g}\right)$$

The only difference between the two mappings is that in the mapping that determines the socially efficient allocation, twice as much weight is put on social gain. Consumers, when choosing between suppliers, trade off technological preferences and social gains. However, social gain is matched by an equally large externality on the other agents in the network. The technological preferences, in contrast, are carried by the agent in their entirety. This explains why the planner puts twice as much weight on social value relative to technological preferences as the market.

For $z_i < 0$, $H(z_i) > 1/2$. Thus, the agent located at z_i obtains more social value by joining the A-network than the B-network. For the same reason, the positive externality of joining the A-network is larger than the positive externality associated with joining the B-network, and it follows that $H^*(z_i) > H(z_i)$ on the entire southern hemisphere. The opposite holds on the northern hemisphere

Put differently, the net externalities associated with increasing $H(z)$ at $z = z_i$ in the market solution $H(z)$ is $\mathbf{g}_A(z_i) - \mathbf{g}_B(z_i)$ where $\mathbf{g}_A(z_i)$ and $\mathbf{g}_B(z_i)$ are evaluated for the equilibrium distribution H. Again observe that the net externality is positive if the marginal agent at z_i has a majority of friends in the A-network. An agent on the southern hemisphere has more friends connected to the A-network than the B-network. Hence if she chooses firm A, the net externality is positive.

8 Prices

In what follows we want to analyze equilibrium prices. From Proposition 4 we have that the pure strategy equilibrium satisfy

$$p_A = p_B = c + \frac{1}{2N_A'(0)}.$$

Since y has support on $[-\infty, \infty]$, in equilibrium, $0 < H(z) < 1$ for all z, which means there are marginal agents for all locations z. We have (see appendix for details)

Lemma 6

$$N_A'(\cdot) = \int \frac{f^z\left(y^m(z)\right)}{1 - 2\mathbf{g}^m(z)} dz \tag{14}$$

where

$$\mathbf{g}^m(z) = \int g(d(z, z_i)) f^z\left(y^m(z_i)\right) dz_i$$

The term $\mathbf{g}^m(z)$ has a clear interpretation. It is the aggregate gain for all the marginal customers around the circle of having one more customer at z. Note that in the intergrand, $f^z\left(y^m(z_i)\right)$ enters multiplicatively, the higher the number of marginal customers at z_i, the higher is the weight on $g(d(z, z_i))$. Note also that (14) has the feature of a "multiplicator". Due to the positive externality, new members make the network more valuable which stimulates even more agents to join, and so on.[8]

Inserting $\mathbf{g}^m(z)$ in (8) immediately gives us our next proposition:

Lemma 7 *In a pure strategy equilibrium, prices are given by*

$$p_A = p_B = c + \frac{1}{2\int \frac{f^z(y^m(z))}{1-2\mathbf{g}^m(z)} dz} \tag{15}$$

Let us discuss how prices depend on the degree of correlation between social location and technological preferences more in detail. In the analysis we assume that assumption 1-3 are satisfied. If social location and the tecnhology preference are uncorrelated, $a = 0$, then $y^m = 1/2$ for all z_i,

[8] It follows that $2\int g(d(z, z_i)) f^z\left(y^m(z_i)\right) dz_i < 1$. To see this, note that since $f(\cdot)$ is single peaked and symmetric around 0, $f(0) \geq f(y)$ for all y. Thus $2\mathbf{g}^m(z) = 2\int \left[g(d(z, z_i)) f\left(y^m(z_i) - az_i\right)\right] dz_i \leq 2\int \left[g(d(z, z_i)) f^{\max}\right] dz_i = 2\mathbf{g} f^{\max} < 1$ by assumption.

and $\mathbf{g}^m(z) = f(0)\mathbf{g}$. Hence, [9]

$$p_A = p_B = c + \frac{1}{2f^z(0)} - \mathbf{g} \tag{16}$$

Now we turn to the effects of lcoal network effects, arising from a higher correlaction. as described above.

Proposition 5 *Equilibrium prices are monotonically increasing in the degree of correlation between social and technological preferences.*

For given values of $y^m(z_i)$, an increase in a increases $|y^m(z_i) - az_i|$ (since the last term has the same sign as the first term). Furthermore, as we show in the appendix, an increase in a increases $|y^m(z_i)|$ for all $z_i \neq 0$. Since $f()$ is hump-shaped around 0 it follows that $f(y^m(z_i) - az_i)$ decreases for all z_i. This in turn also reduces $g^m(z_i)$, and it follows from (15) that prices increases.

Intuitively, as a increases, the marginal consumers tend to be more extreme. For the A-network, the market share becomes even bigger for $z_i < 0$ and smaller for $z_i > 0$, and the opposite for the B network. In the tails, the density of agents is lower, hence there are fewer marginal customers. This weakens competition and reduces prices.

As above, write $g(z) = k\bar{g}(z)$, and consider an increase in k. When location and technological preferences are uncorrelated ($a = 0$), the price is given by (16), and an increase in k implies that prices fall. With a positive correlation, an increase in k also increases $|y^m(z_i)|$, see lemma 3. As we have seen, a higher $|y^m(z_i)|$ reduces the number of marginal customers $f(y^m - az)$ as well as $\mathbf{g}^m$, and from (15) we know that this tends to increase prices. The total effect is therefore ambigous. However, as our next proposition shows, increased network effects may actually increase prices. As above we write $g(z) = k\bar{g}(z)$, and without loss of generality we normalize $\bar{\mathbf{g}}$ to 1.

Proposition 6 *Let $g(z) = k\bar{g}(z)$. Then a higher k may imply a higher equilibrium prices.*

In the appendix we give an example. In the example, network externalities are "pure local", in the sense that $g(z_i)$ is concentrated around z_i, i.e., that an agent only cares about the choice of network of the agents that are socially very close to him.

[9] Note that the equilibrium price p approaches marginal cost as $2f^z(0)$ approaches $\mathbf{g}$. This illustrates the possibility that an equilibrium in pure strategies ceases to exist if competition is fierce. If $2f^z(0)$ approaches $\mathbf{g}$ and the f^z-distribution has a long tail (with $f^z(z)$ strictly below $f^z(0)$ (in the tail) the single firm has an incentive to deviate, and charge a high price, serving customers with a very strong preference for its network, and obtain a positive margin.

Proof: All we need to do in order to prove the proposition is to give an example in which an increase in k leads to higher prices. We normalize $\bar{\mathbf{g}}(z)$ to 1. Consider the limit at which the support of $g(z_i)$ collapses to z_i, in which case we can write $\mathbf{g}^m(z) = \int g(d(z, z_i)) f^z\left(y^m(z_i)\right) dz_i = kf^z(y^m(z_i))$. Let f be uniform and equal to 1 on an interval $[-1/2 + \varepsilon, 1/2 - \varepsilon]$, with the rest of the probability mass, 2ε, are continously distributed outside this interval with unbounded support. We analyze the limit case in which $\varepsilon \to 0$, so that $f^m(y^m) \to 0$ outside the interval. Furthermore, let $a > 1$. From footnote 7 we know that at the interval at which $f = 1$, $y^m(z_i) = -\frac{2kaz_i}{1-2k}$. The interval at which $f = 1$ is thus given by $|y^m(z_i)| < 1/2$, i,e, $|z_i| < \frac{1-2k}{4ka}$. The derivative wrt k evaluated at $k = 0$ is thus $-\frac{1}{4a}$. Let $\kappa(k) = \frac{1-2k}{4ka}$. Then $\int f^m(y^m(z)dz \equiv 2\kappa$. Now consider the denominator in (15), repeated for convenience

$$\rho(k) = 2\int \frac{f^z(y^m(z_i))}{1 - 2kf^z(y^m(z_i))} = \frac{2\kappa}{1 - 2k}$$

It is sufficient to show that this is decreasing at $k = 0$. Taking derivatives give

$$\begin{aligned} \rho'(0) &= 4(\kappa'(k) + 2) \\ &= 2(-\frac{1}{4a} + 2) \end{aligned}$$

which is negative for $a < \frac{1}{8}$.

9 Endogenous agent heterogeneity

Differences in preferences between marginal and average agents may give rise to distortions. This was first explored in Spence's (1975) model of a monopolist's choice of quality. If marginal and average consumers value quality differently, the quality level chosen by the monopolist will not be socially optimal.

Local network externalities, in contrast to global externalities, give rise to a difference between marginal and average agents in a network, because the former on average obtain less utility from interacting than the latter. This may lead to additional distortions that exacerbate the composition inefficiencies analyzed above. In order to simplify the analysis we assume that $F^z(y) = F(y - az)$ with the restrictions on $F()$ laid out above. In the analysis we will draw heavily on the following result, which we refer to as a corollary since it follows almost directly from 1.

Corollary 2 *Suppose $H^1 \in D$ is more concentrated than $H^0 \in D$, and let $\mathbf{g}_A^1(z_i)$ and $\mathbf{g}_A^0(z_i)$ the assocaited numbers of friends in the $A-$network. Then $\mathbf{g}_A^1(z_i) > \mathbf{g}_A^0(z_i)$ for $z_i < 0.$. Furthermore, $\int \mathbf{g}_A^1(z_i)H^1(z_i)dz_i > \int \mathbf{g}_A^0(z_i)H^0(z_i)dz_i$*

The corrolary state that the more concentrated is H, the more friends do individuals located at $z_i < 0$ have in the A network, and the higher is the total number of friends the customers of the $A-$have in that network. In total, each agent has $\mathbf{g}$ friends, divided on the two networks. If H is uniform and equal to $1/2$, then the total measure of friends in the network is $1/2 * \mathbf{g}/2 = \mathbf{g}/4$. The more concentrated is H, the higher is $\mathbf{g}_A$ and the higher is the total number of friends in the network. Conversely, the lower is the total number of friends the agents in the A-network have in the B-network.

9.1 Compatibility

We will now discuss firms' incentives to undertake investments in order to make networks one-way compatible. Thus, network A may give its members (improved) access to network B by undertaking an investment. Let $\theta_A \leq 1$ denote the degree to which agents in network A can utilize network B, and write the cost of compatibility as an increasing an convex function $C(\theta_A)$, with $C(0) = C'(0) = 0$ and $\lim_{c \to 1^-} C'(1) = \infty$. We only include connection pricing (no two-part tariffs). The degree of compatibility is set independently and simultaneously by the two firms at stage 1, together with prices p_A and p_B. In other respects the timing is unchanged.

We assume that compatibility from the A-network to the B-network only benefits the consumers in the A-network (consistent with the assumption above that only the caller receives utility). The utilities of an agent (y_i, z_i) in network A and and B, respectively, are given by $u^A = \beta + \mathbf{g}_A(z_i) + \theta_A \mathbf{g}_B(z_i) - y_i - p_A$ and $u^B(y_i, z_i) = \beta + \mathbf{g}_B(z_i) + \theta_B \mathbf{g}_A(z_i) - p_B$. The marginal consumer in the A-network is characterized by (recall that $\mathbf{g}_A + \mathbf{g}_b = \mathbf{g}$)

$$y^m(z_i) = 2\mathbf{g}_A(1 - \frac{\theta_A + \theta_B}{2}) + (p_B - p_A) - (1 - \theta_A)\mathbf{g}$$

The distribution $H(z)$ is thus defined by the fixed point to the mapping Γ^C defined as

$$\Gamma^C H(z_i) = F\left(2\left(1 - \frac{\theta_A + \theta_B}{2}\right)\int g(d(z, z_i))H(z)dz + p_B - p_A - (1 - \theta_A)\mathbf{g} - az_i\right)$$

Network A's net profit equals

$$\pi_A = p_A \int H(z)dz - C(\theta_A)$$

In the appendix we show that the firms will choose degree of compatibility such that the marginal customers' valuation of compatibility equals marginal costs. Recall from the last section that the marginal customers on average have half of their friends in the other network. The first order condition for θ_A is thus

$$C'(\theta_A) = \frac{\mathbf{g}}{4} \tag{17}$$

The socially efficient degree of compatibility (contingent on equal market shares), by contrast, maximizes welfare W defined by (12) less the costs $C_A(\theta_A) + C_B(\theta_B)$:

$$W = \int [\mathbf{g}_A(z_i) + \theta_A \mathbf{g}_B(z_i)\mathbf{g}_B(z_i) + \theta_B \mathbf{g}_A(z_i)]H(z_i) + Y - C_A(\theta_A) - C_B(\theta_B):$$

Maximizing W w.r.t. θ_A at $H = H^*$ (the socially optimal distribution) gives the first order condition

$$C'_A(\theta_A) = \int \mathbf{g}_B(z_i)H^*(z_i)dz_i \tag{18}$$

Recall that $\int \mathbf{g}_B(z_i)H^*(z_i)dz_i = \mathbf{g}/2 - \int \mathbf{g}_A(z_i)H^*(z_i)dz_i$. From Corollary 2 we know that $\int \mathbf{g}_A(z_i)H^*(z_i)dz_i > 1/2$. Hence right-hand side of (18), the total number of "friends" that the members of network A have in network B given the optimal H^*, is less than $\mathbf{g}/4$.

Proposition 7 *The firms have too strong incentives to make the networks (one-way) compatible.*

The planner is concerned with the average customers' utility from compatibility. The network owner, by contrast, cares about the marginal customer's utility from compatibility. Although the average customer in the A network has more friends in that network than in the B network, this is not the case for the average marginal consumer. To understand this, consider two social locations z_i and $-z_i$, $z > 0$. Although the A network has more consumers at $-z$ than at z, the number of marginal consumers $f(y^m(z) - az)$ is the same at the two points. Hence symmetry implies that the average number of friends among the marginal consumers is $\mathbf{g}/2$. This result emerges despite the fact that there are no externalities associated with compatibility in itself, as compatibility is one-way. [10]

[10] Farrel and Saloner (1992) find in a model with global network externalities that firms choose an optimal level of compatibility. Our result shows that their result is not robust when allowing for local network externalities.

The comparison above is between compatibility in the market solution and first best compatibility. It follows directly if the planner was setting the compatibility level given the market distribution H, the planner would still set a lower θ than the market solution. If the planner decides θ based on the market distribution H instead of H^*, the right-hand side of (18) would still be less than $\mathbf{g}/2$ and we would still have overinvestment in compatibility. In addition, increasing θ_A has a negative effect on composition efficiency, since it attracts agents that communicate intensively with the other network (that is types $z_i > 0$) and punish agents with most of their friends in the A-network (types $z_i < 0$). Hence, a high level of compatibility makes the equilibrium distribution $H(z)$ flatter. However, we have already seen that the efficient distribution $H^*(e)$ is steeper than the equilibrium distribution H. Hence, in the constrained efficient solution (where the planner sets the level of compatibility but nothing else), the planner would reduce compatibility further in order to obtain a more efficient composition of consumers on networks.

9.2 Usage intensity

In this subsection we assume that consumers, when connected to a network, choose how much to use it. We use communication platforms as our example. The argument could also be applied to platforms where agents may choose how many applications to buy as well as to clubs.

We assume that the utility a consumer obtains from usage within a relationship is endogenous and given by $\omega(x)$, where x is usage. Firms compete by offering two-part tariffs (p_j, q_j), $j = A, B$, where p is a fixed fee and q is the cost of using the network. The net surplus $v(q_A)$ per friend for a consumer in network A is

$$v(q_A) = \underset{x}{Max}[\omega(x) - q_A x]$$

We write the optimal usage as a function of q_A, $x(q_A)$. Note that $x(q_A) \equiv -v'(q_A)$.

Firms advertise a pair (p_j, q_j). The utility for a agent of joining the A network is $v(q_A)\mathbf{g}_A(z_i) - y_i - p_A$ and of joining the B network $v(q_B)\mathbf{g}_B(z_i) - p_B$. By reasoning as above it follows that for given prices, the equilibrium distribution $H^x(z)$ is the fixed point to the mapping Γ^x given by

$$\Gamma^x H(z_i) = F\left((v(q_A) + v(q_B)) \int g(d(z, z_i))H(z)dz + p_B - p_A - v(q_B)\mathbf{g} - az_i\right) \tag{19}$$

Note that for given q_A and q_B, $v(q_A)$ and $v(q_B)$ are constants, hence we can show existence and uniqueness of the fixed point in exactly the same way as above.

Define $\mathbf{G}_A \equiv \int \mathbf{g}_A(z_i)H(z_i)dz_i$ as the total number of customers in the network.[11] From corollary 2 it follows that in a symmetric equilibrium, $\mathbf{G}_A > \mathbf{g}/4$. The highest possible value of $\mathbf{G}_A$ is 1/2.[12] The profit of firm A is given by

$$\pi_A = (p_A - c)N_A + (q_A - c_x)x(q_A)\mathbf{G}_A \tag{20}$$

It follows that $x(q_A)\mathbf{G}_A$ shows aggregate usage of the network, while $(q_A - c_x)$ is the mark-up per unit of usage. We only consider symmetric equilibria, where each of the firms has half of the market. In the appendix we derive the profit maximizing mix of usage price and fixed fee, given that the market share is 1/2. It follows that the first order condition for q_A can be written as

$$\frac{1}{2}[1-\gamma]x(q_A) + (q_A - c_x)x'(q_A) + \frac{x(q_A)(q_A - c_x)el_q\mathbf{G}_A}{q_A} = 0 \tag{21}$$

where

$$\gamma := \frac{\mathbf{g}/4}{\mathbf{G}_A} < 1$$

and $el_q\mathbf{G}$ is the elasticity operator. The variable γ represents the total number of friends that the marginal customers have in the network relative to the number of friends that all the consumers in the network have.

The denominator shows the total number of "friends" in the network, which is also the average number since by normalization each network has a measure of 1 customers. With pure global network externalities ($a = 0$), $\gamma = 1$. However, if $a > 0$, then $\gamma \in (1/2, 1)$. The first term in (21) thus represents rent extraction from the inframarginal customers. Since inframarginal customers on average have higher usage intensity than marginal customers, increasing the usage price increases total payments from existing customers, even though the fixed price p_A is reduced so that the market share of the firm stays constant. The second term in (21) is self-explanatory. The last term shows the change in incomes from usage fees caused by changes in the composition of the network. In the appendix we show that $el_q\mathbf{G}_A < 0$: A higher usage price hurts the marginal agents with many friends in its own network (z low) more than those with a few (z high). A higher q_A thus

[11] Each pair of friends counts as two connections, as person i is friends with person j and person j is friends with person i.

[12] This is obtained in the limit when i) the market is divided such that all customers in the southern (northern) hemisphere belong to network A (B) and all a customers's friends have a social position that is arbitrarily close to his.

implies that H becomes flatter, and hence that total traffic falls (even though the market share stays constant).

However, with marginal cost pricing, $q_A = c_x$, the last term in (21) is zero. Hence with marginal cost pricing, the left-hand side of (21) is strictly positive as long as $\gamma < 1$. The next proposition is thus immediate

Proposition 8 *The firms set the usage price q_k, $k = A, B$ above marginal cost. Thus, the usage price exceeds the price level that induces a static first best level of traffic represented by marginal cost pricing (provided that $\gamma < 1$).*

This finding contrasts with the standard result that a two-part tariff induces marginal cost pricing on usage and therefore efficient usage in the standard model without local network externalities (Farrel and Saloner 1992). Local externalities create agent heterogeneity, and since marginal customers on average have lower usage than inframarginal customers, traffic price can be used as a rent extraction device. The firm thus trades off efficiency and rent extraction for the inframarginal ("high-type") agents.

The network owner prices internal traffic as if she had some degree of market power, where the degree of market power is captured by the relative deviation between the marginal and the average intensity of exchange. With global network externalities, symmetry between agents prevails (hence $\gamma = 1$), which means that the network adopts marginal cost pricing. In the appendix we show that γ decreases as the spread of g decreases.

A higher usage price hurts agents with many friends in the network (z low) more than those with a few friends in their network (z high). As we show in the appendix, a higher usage price makes the equilibrium distribution less concentrated. Hence excessive usage pricing moves the equilibrium distribution further away from the composition efficient distribution. In a constrained efficent solution, where the planner can decide on q_A but nothing else, the planer would set the usage price above marginal costs in order to improve on the equilibrium distribution H. The distortions in usage pricing thus accesarbate the composition inefficiency created by social externalities.

10 Concluding remarks

Network externalities are important in several markets, particularly those related to information and communication technologies. In the economics literature, the focus has been on global network externalities, where network effects are related solely to size. In the present paper we argue that the network effects not only work through the size of the customer base, but also through its composition, i.e., the identity and/or attributes of the customers in the customer base, in particular their exogenously given relationships to each other. We refer to this as local network externalities.

We propose a way of modeling local network externalities, which is sufficiently rich to capture the main attributes of network composition and still sufficiently simple to make the analysis tractable, and which embodies global externalities as a special case. We do this by using a two-dimensional spatial model. Consumers have a location in a social space, and interact mostly with people located closely to them in this space and less intensely with people further away. This assumption is consistent with dominant sociological findings on the structure of social networks (c.f. Granovetter 2005). In addition, consumers' technological preferences are represented by a location in technological space. Finally, the consumers' location in the two spaces may be correlated in the sense that if two agents are close in the social space they are also likely to be close in the technological space.

Two firms that are horizontally differentiated in technology compete for customers. We show that as long as social preferences do not dominate technological preferences, the model has a unique equilibrium. The equilibrium has several interesting properties. First, a higher correlation between technological and social preferences lead to lower competitive pressure and higher prices. Second, the allocation of consumers on networks is not efficient, as there is a social externalities associated with the choice of network that the customers do not take into account. Third, local network externalities give rise to differences between average and marginal consumers, which leads to inefficiently high usage prices and excessive levels of (one-way) compatibility.

11 Appendix

11.1 Proofs related to existence

Proof of proposition 1

In this proof, it is convenient to relable the location. Define the location on $[0,1]$, with $z=0$ at the south pole, and increasing clockwise to $1/2$ at the north pole, and to 1 again at the south pole. It follows that $d(z_1,z_2)=\min[|z_2-z_1|,1/2-z_1-z_2]$. The distance between two points on the cricle is unalterned, and the model is isomorphic to the model presented in the text.

Let C denote the subset of continous functions $C[0,1]$ that are bounded below by 0 and above by 1. Clearly D is bounded and convex. Furthermore, Γ is a continous mapping from C to C. We will show that our assumptions on the densities of F imply that the family of functions ΓH is equicontinuous, in which case Schauder's fixed-point theorem applies. For any given $\varepsilon>0$, we have to show that there exists a $\delta>0$ such that $|\Gamma H(z_1)-\Gamma H(z_0)|\leq\varepsilon$ for all $|z_1-z_0|<\delta$ and for all $H\in C$. Adding and subtracting $F^{z_1}(\beta+\int g(d(z,z_0))H(z)dz-p)$ and using the triangle inequality give

$$\begin{aligned}
|\Gamma H(z_1)-\Gamma H(z_0)| &= |F^{z_1}(\beta+\int g(d(z,z_1))H(z)dz-p)-F^{z_0}(\beta+\int g(d(z,z_0))H(z)dz-p)| \\
&\leq |F^{z_1}(\beta+\int g(d(z,z_1))H(z)dz-p)-F^{z_1}(\beta+\int g(d(z,z_0))H(z)dz-p)| \\
&\quad +|F^{z_0}(\beta+\int g(d(z,z_0))H(z)dz-p)-F^{z_1}(\beta+\int g(d(z,z_0))H(z)dz-p)|
\end{aligned}$$

Now

$$\begin{aligned}
& |F^{z_1}(\beta+\int g(d(z,z_1))H(z)dz-p)-F^{z_1}(\beta+\int g(d(z,z_0))H(z)dz-p| \\
\leq\ & f^{\max}|\int[g(d(z,z_1))-g(d(z,z_0))]H(z)dz| \\
\leq\ & f^{\max}g(0)|z_1-z_0|
\end{aligned}$$

Furthermore,

$$\begin{aligned}
& |F^{z_0}(\beta+\int g(d(z,z_0))H(z)dz-p)-F^{z_1}(\beta+\int g(d(z,z_0))H(z)dz-p)| \\
\leq\ & f^{z}|z_1-z_0|
\end{aligned}$$

Hence

$$|\Gamma H(z_1) - \Gamma H(z_0)| \leq (f^{\max} g(0) + f^z)|z_1 - z_0|$$

for all H, which is less than ε if $|z_1 - z_0| \leq \delta = \frac{\varepsilon}{f^{\max} g(0)+f^z}$.

Proof of Proposition 2

We have to show that Γ is a contraction. To this end, let H_1 and H_2 denote two arbitrary distribution functions. Then

$$\begin{aligned}
& \sup_{z_i} |\Gamma H_1(z_i) - \Gamma H_2(z_i)| \\
= & \sup_{z_i} |F^z(\beta + \int g(d(z, z_i))H_1(z)dz - p) - F^z(\beta + \int g(d(z, z_i))H_2(z)dz - p)| \\
\leq & f^{\max}\mathbf{g} \sup_{z_i} |H_1(z_i) - H_2(z_i)| \\
< & \sup_{z_i} |H_1(z_i) - H_2(z_i)|
\end{aligned}$$

since, by assumption, $f^{\max}\mathbf{g} < 1$. Hence Γ is a contraction.

Proof of continuity of $N(p_B - p_A)$

Let $\alpha = 2f^{\max}\mathbf{g} < \mathbf{1}$. Consider a price change of $\Delta p = p_A - p_B$. It follows that $|\Gamma(H + \Delta p) - \Gamma H| = F^z(2\int g(d(z, z_i))H_1(z)dz - p) - F^z(2\int g(d(z, z_i))H_2(z)dz - p - \Delta p)| \leq 2f^{\max}\Delta p$. Without loss of generality, suppose $\Delta p > 0$. Let $H_0 = \min[H + 2f^{\max}\Delta p, 1]$. Then $H^1(z) = \Gamma H_0(z) < H^0 + \alpha\Delta p$. Using the operator repeatedly it follows that $|\Gamma(H + \Delta p) - \Gamma H| \leq \frac{f^{\max}\Delta p}{1-\alpha}$, which converges to zero as $\Delta p \to 0$. Continuity follows.

11.2 Proofs related to the section "Equilibrium distrubutions"

Proof of Lemma 1

Since the mapping is symmetric around $z = 0$, it follows that $H^1(z_i)$ is a symmetric distribution function. We have to show that H^1 is monotone. First we want to show that $\int g(d(z_i, z))H^0(z)dz$

is decreasing in z_i. Taking derivative with respect to z_i gives (this is innocous, as it does not require that $H(z)$ is continuous)

$$\frac{d}{dz_i}\int g(d(z_i,z))H^0(z)dz = \int g'(d(z_i,z))\frac{\delta d(z,z_i)}{\delta z_i}H^0(z)dz$$

Since g is symmetric, we can rewrite this as

$$\int g'(d(z_i,z))\frac{\delta d(z,z_i)}{\delta z_i}H_0(z)dz = \int_{x=0}^{1/2} g'(x)[H^0(z^+(x)) - H^0(z^-(x))dx$$

where $z^-(x)$ is the location obtained by going clockwise x units and $z^+(x)$ the location obtained by going counter-clockwise x units. Since $H^0(z)$ is symmetric and monotonically decreasing in z_i^j, $j \in \{e,w\}$ when going from south to north, it follows that $H^0(z^+(x)) \leq H^0(z^-(x))$ for all $x \in [0,1]$, with strict inequality if the bounds do not bind. It follows that $\int 2g(d(z_i,z))H^1(z)dz - \mathbf{g}$ is decreasing in z_i, and strictly so i the bounds don't bind everywhere.

Next we want to show that $H^1(z)$ is strictly decreasing. Let z_i and z_j be two arbitrary values of z, with $z_i < z_j$. Then

$$\begin{aligned}
\Gamma H_0(z_j) &= F^{z_j}\left(\int 2g(d(z,z_j))H_0(z)dz - \mathbf{g}\right) \\
&\leq F^{z_j}\left(2\int g(d(z,z_i))H_0(z)dz - \mathbf{g}\right) \\
&< F^{z_i}\left(2\int g(d(z,z_i))H_0(z)dz - \mathbf{g}\right) \\
&= \Gamma H_0(z_i)
\end{aligned}$$

Hence ΓH_0 is strictly decreasing in z.

Suppose $H^1 = \Gamma H^0$ is more concentrated than H^0. Let $H^2(z_i) = \Gamma H^1(z_i)$. We want to show that H^2 is more consentrated than H^1. Now

$$\begin{aligned}
H^2(z_i) &= F^{z_i}\left(2\int g(d(z,z_i))H_1(z)dz - \mathbf{g}\right) \\
&= F^{z_i}(2\int g(d(z,z_i))H_0(z)dz - \mathbf{g}) + 2\int g(d(z,z_i))(H_1(z) - H_0(z))dz) \\
&= F^{z_i}(y_1^m(z_i) + 2\int g(d(z,z_i))(H_1(z) - H_0(z))dz)
\end{aligned}$$

where $y_1^m = y_1^m(z_i)$ is the technology preference of the marginal consumer at z_i. It is thus sufficient to show that $\int g(d(z,z_i))(H^1(z) - H^0(z))dz > 0$ for $z_i < 0$. Let $A(z) \equiv H^0(z) - H^1(z)$. Since

H_1 and H_2 are symmetric distribution functions, we have that $A(z) > 0$ for $z < 0$ and that $A(z) = -A(-z)$. Suppose $z_i < 0$. Let x denote the distance from equator to any point on the circle. Then we can write

$$\begin{aligned} & \int_\Omega g(d(z, z_i))(H^1(z) - H^0(z))dz. \\ = & \int_0^{1/2} A(z)[g(|z - z_i|) - g(z_i)]dz \\ > & \ 0 \end{aligned} \tag{22}$$

Proof of lemma 2: Concavity/convexity properties of H

We want to show that H is concave on $z_i^w \in [-1/4, 0]$. Due to symmetry it then follows that H is concave on $z_i^e \in [-1/4, 0]$ and concave on the complementary part of the circle. Let $z_2 < z_1 < 0$ and $\bar{z}$ the average. Let $\Delta = |z_2 - z_1|$. We assume that Δ is small.

First, define $H_0(z_i)$ as $H_0(z_i) = 1/2$ for all z_i. Consider $H_1 = \Gamma H_0$. Since the network effects are equally strong in both networks, only the technological preferences matter, and $y^m(z_i) = 0$ for all z_i. It follows that $H(z_i) = F(-az_i)$. By assumption $F(y)$ is concave for $y > H$. It follows that $H_1(z_i)$ is concave on $z_i^w \in [-1/4, 0]$.

Now suppose $H_k(z_i)$ is concave on $z_i^w \in [-1/4, 0]$ and $z_i^w \in [-1/4, 0]$ and convex on the complementary part of the circle. Recall that

$$y^m(z_i) = 2\int g(z, z_i)H_k(z)dz - \mathbf{g}$$

It follows that

$$\begin{aligned} \frac{y^m(z_1) + y^m(z_2)}{2} &= 2\int \frac{g(z, z_1) + g(z, z_2)}{2} H_k(z)dz - \mathbf{g} \\ & \quad 2\int g(z, \bar{z}) \frac{H(z - \Delta/2) + H(z + \Delta/2)}{2} - \mathbf{g} \end{aligned}$$

Note that $\frac{H(z-\Delta/2)+H(z+\Delta/2)}{2} < H(z)$ for $z < 0$ (on the concave part) while $\frac{H(z-\Delta/2)+H(z+\Delta/2)}{2} > 0$ for $z > 0$ (on the convex part).

Now we rewrite the integral. We start from equator (say at the west side). Let x denote the distance from equator. Let $z(x^-)$ the position when goint to the south ($z^w = -x$ on the western hemisphare and $z^e = -1/4 + x$ when on the eastern). Let $z(x^+)$ denote the position when going to the north $z^w = x$ when on the western hemisphere and $z^w = 1/4 - x$ when on the eastern. Then

$$
\begin{aligned}
& \int_\Omega g(z,\bar{z})\frac{H(z-\Delta/2)+H(z+\Delta/2)}{2} \\
= & \int_0^{1/2} [g(d(x^-,\bar{z}))\frac{H(z(x^-)-\Delta/2)+H(z(x^-)+\Delta/2)}{2} + g(d(x^-,\bar{z}))\frac{H(z(x^-)-\Delta/2)+H(z(x^-)+\Delta/2)}{2}]dz \\
= & \int_0^{1/2} [g(d(x^-,\bar{z}))(H(x^-)+\xi(x^-)) + g(d(x^-,\bar{z}))(H(x^+)+\xi(x^+))]dz
\end{aligned}
$$

Given the concavity/convexity properties of H_k as well as symmetry, it follows that $\xi(x^-) > 0$ and that $\xi(x^-) = -\xi(x^+)$. Since $g(d(x^-,\bar{z})) > g(d(x^+,\bar{z}))$ for all $x \in (0, 1/2)$ it follows that

$$\int_\Omega g(z,\bar{z})\frac{H(z-\Delta/2)+H(z+\Delta/2)}{2} < \int_\Omega g(z,\bar{z})H(z)dz$$

and hence that $y^m(\bar{z}) > \frac{y^m(z_1)+y^m(z_2)}{2}$. Since F is concave for $y^m > 0$ the result follows.

CHRISTIAN: FIGUR? The opposite holds for $z > 0$. The claim thus follows.

Proof of lemma 3 and 4

We first prove lemma 3. Let k^h and k^l denote two values of k, $k^h > k^l$. Let H^h, H^l and Γ^h, Γ^l denote the corresponding equilibrium distributions and equilibrium mappings, respectively. Finally, let $y_l^m(z_i)$ denote the technology preference of the marginal customer at z_i given that $k = k^l$. Define

$$
\begin{aligned}
H_1^h(z_i) &= \Gamma^h(H^l)(z_i) \\
&= F^{z_i}(\int k^h H^l \bar{g}(d(z,z_i)dz - k^h \mathbf{\bar{g}}) \\
&= F^{z_i}(\frac{k^h}{k^l}\int k^l H^l \bar{g}(d(z,z_i)dz - k^l \mathbf{\bar{g}}) \\
&= F^{z_i}(\frac{k^h}{k^l} y_l^m)
\end{aligned}
$$

It follows directly that $H_1^h(z_i)$ is more concentrated than $H^l(z_i)$. Since Γ^h is a contraction, we know that $H^h = \lim_{n\to\infty} \Gamma^{hn} H^l$, (where Γ^{hn} is the operator Γ^h applied n times), hence it follows from lemma 1 that H^h is steeper than H^l. This completes the proof of lemma 3.

We will then prove lemma 4, and proceed in the same way. We use the same notation. Let a^h and a^l denote two values of a, $a^h > a^l$. Analogous with the above notation, let H^h, H^l and Γ^h, Γ^l denote the corresponding equilibrium distributions and equilibrium mappings with $a = a^h$ and $a = a^l$, respectively. Finally, let $y_l^m(z_i)$ denote the technology preference of the marginal customer at z_i given that $a = a^l$. Define

$$\begin{aligned} H_1^h(z_i) &= \Gamma^h(H^l)(z_i) \\ &= F(\int H^l g(d(z, z_i)dz - \mathbf{g} - a^h z) \\ &= F(y_l^m - (a^h - a^l)z) \end{aligned}$$

Since $(a^h - a^l)z$ is negative for $z < 0$ and positive for $z > 0$, it follows imediately that $H_1^h(z_i)$ is more concentrated than H^l, and by applying lemma 1 that H^h is more concentrated than H^l. This completes the proof.

Proof of Lemma 5

Proof that a more concentrated g gives a more concentrated H

Consider two utility functions $\dot{g}_1$ and g_2, and suppose g_2 is more concentrated than g_1. Let H_1 denote the equilibrium distribution function associated with g_1. Consider $\bar{H} = \Gamma^{g_2} H_1$. We want to show that $\bar{H}$ is more concentrated than H_1. Then it follows from lemma ??? that H_2 is more concentrated than H_1.

Recall that

$$y^m(z_i) = 2 \int g(d(z, z_i) H_1(z_i) dz_i - \mathbf{g}$$

Note that we can write

$$\int g(d(z, z_i) H_1(z_i) dz_i = \int_{x=0}^{1/2} g(x)[H(z^+(x) + H(z^-(x))]dx$$

Write $h(x) = [H(z^+(x) + H(z^-(x))]$. Since H is concave for $z < 0$ and convex for $z > 0$ it follows that $h(x)$ is decreaseing in x, and hence that $\int_{x=0}^{1/2} g(x)h(x)dx$ decreases when $g(x)$ becomes more concentrated. This completes the proof.

11.3 Proofs regarding efficiency

Maximizing and minimizing social value.

With two symmetric networks this is equivalent to maximizing V_A with respect to the distribution $H(z)$ subject to $\int H(z)dz = 1$, that is

$$\max_{H(z_i)} \iint g(z_i - z)H(z)H(z_i)dzdz_i \quad \text{s.t.} \quad \int H(z_i)dz_i = 1$$

with the associated Lagrangian

$$L = \int \left[\int g(z_i - z)H(z)dz - \lambda \right] H(z_i)dz_i$$

Point-wise maximization yields the first order condition

$$\begin{aligned}
\int g(z_i - z)H(z)dz - \lambda &> 0 \rightarrow H(z_i) = 1 \\
\int g(z_i - z)H(z)dz - \lambda &< 0 \rightarrow H(z_i) = 0 \\
\int g(z_i - z)H(z)dz - \lambda &= 0 \rightarrow H(z_i) \text{ undetermined}
\end{aligned}$$

Obviously there are two solutions satisfying the first order conditions, either $H(z) = 0.5$ all z, or $H(z) = 1$ for all $z \epsilon [z', -(1 - z')]$ where z' is arbitrary, and $H(z) = 0$ otherwise.[13] The two solutions are referred to as the maximum and minimum solutions respectively.

First order conditions

Recall that the welfare function is given by (from 12)

$$W = \int [\mathbf{g}_A(z)H(z) + \mathbf{g}_B(z)(1 - H(z)) + Y(z)]dz$$

We maximize (12) point-wise with respect to $H(z_i)$. First we characterize the derivative of $Y(z)$ with respect to $H(z_i)$. Since $F^z(y^m(z)) = H(z)$, implicit derivation gives $dy^m/dH = 1/f^z(y^m(z))$. Hence, from (11)

$$\frac{dY(z_i)}{dH(z_i)} = -y^m(z)$$

[13] Observe from the first order conditions that the number of friends in the A network, $\int g(z_i - z)H(z)dz$, must be equal for all z_i at which $H(z_i)$ is strictly between 0 and 1. Then it follows trivially that H can be interior on an interval only if $H = 0.5$ everywhere.

Then note that

$$\begin{aligned}
& \frac{d}{dH(z_i)} \int [\mathbf{g}_A(z)H(z) + \mathbf{g}_B(z)(1-H(z))]dz \\
= \; & \mathbf{g}_A(z_i) - \mathbf{g}_B(z_i) + \int [\frac{d\mathbf{g}_A(z_i)}{dH(z_i)} H(z) + \frac{d\mathbf{g}_B(z_i)}{dH(z_i)}(1-H(z))]dz \\
= \; & \mathbf{g}_A(z_i) - \mathbf{g}_B(z_i) + \int [g(d(z_i, z)H(z) - g(d(z_i, z)(1-H(z))]dz \\
= \; & 2\mathbf{g}_A(z_i) - 2\mathbf{g}_B(z_i)
\end{aligned}$$

To get from the second to the third equation we used that a one unit increase in H on an interval dz around z_i increases the social value for an agent at z_j if joining the network by $g(d(z_i, z_j))dz$ units). To go from the third to the last equations we used the definitions of $\mathbf{g}_a$ and $\mathbf{g}_b$. The first order condition for maximum is thus that $2\mathbf{g}_A(z_i) - 2\mathbf{g}_B(z_i) - y^m(z_i)) = 0$. Inserting from $\mathbf{g}_A(z) + \mathbf{g}_B(z) = \mathbf{g}$, and set the derivative to zero gives

$$2\left[2\mathbf{g}_A(z_i) - \mathbf{g}\right] - y^m(z_i) = 0$$

Hence

$$\begin{aligned}
H^*(z) &= F^z(y^m(z)) \\
&= F^z\left(2\left[2\mathbf{g}_A(z) - \mathbf{g}\right]\right) \\
&= F^z\left(2\left[2\int g(d(z, z_i))H^*(z)dz - \mathbf{g}\right]\right)
\end{aligned}$$

as stated in the text.

11.4 Proof related to the section "Prices"

Proof of Lemma 6 (in main text)

The definition of Γ given by (4) reads

$$H_1(z_i) = F\left(2\int g(d(z, z_i))H_0(z)dz - \mathbf{g} + p_B - p_A - az_i\right) = F\left(y^m(z_i) - az_i\right)$$

As Γ is a contraction mapping with modulus $\alpha < 1$ (see proof of Proposition 1), we can apply the method of successive appoximations. Note that a partial reduction in p_A has an immediate impact on market shares in addition to an infinite sequence of derived impacts. The aggregate impact on firm A's market share is then the sum of this infinite sequence of small changes.

Refer to $dH^0(z_i)$ as the direct initial impact on market share at location z_i due to a *reduction* in p_A,

$$dH^0(z_i) = f\left(y^m(z_i) - az_i\right) dp_A$$

which is proportional to the density of marginal consumers at location z_i. The aggregate initial effect on firm A's market share is then the integral over all locations

$$\int dH^0(z_i)dz_i = \left[\int f\left(y^m(z_i) - az_i\right) dz_i\right] dp_A$$

Consider then the derived first round impact that follows from network externalities. For each z, insert $dH_0^0(z)$,

$$\begin{aligned} dH^1(z_i) &= 2f\left(y^m(z_i) - az_i\right) \int g(d(z, z_i))dH^0(z)dz \\ &= 2f\left(y^m(z_i) - az_i\right) \left[\int g(d(z, z_i)) f\left(y^m(z) - az\right) dz\right] dp_A \\ &= 2f\left(y^m(z_i) - az_i\right) \mathbf{g}^m(z_i)dp_A \end{aligned}$$

where $\mathbf{g}^m(z_i)$ is defined in (14) in the main text.

The integrated effect on A's market share is then:

$$\int dH^1(z_i)dz_i = \left[\int 2\mathbf{g}^m(z_i) f\left(y^m(z_i) - az_i\right) dz_i\right] dp_A$$

The second round effect is

$$\begin{aligned} dH^2(z_i) &= 2f\left(y^m(z_i) - az_i\right) \int g(d(z, z_i))dH^1(z)dz \\ &= 4f\left(y^m(z_i) - az_i\right) \left[\int g(d(z, z_i)) f\left(y^m(z) - az\right) \mathbf{g}^m(z)dz\right] dp_A \end{aligned}$$

We integrate the second round effect

$$\begin{aligned} \int dH^2(z_i)dz_i &= 4\left[\int\int g(d(z, z_i)) f\left(y^m(z_i) - az_i\right) f\left(y^m(z) - az\right) \mathbf{g}^m(z)dzdz_i\right] dp_A \\ &= \left[\int \left[2\mathbf{g}^m(z)\right]^2 f\left(y^m(z) - az\right) dz\right] dp_A \end{aligned}$$

And generally, the $k't$ order effect is

$$\int dH^k(z_i)dz_i = \left[\int \left[2\mathbf{g}^m(z)\right]^k f\left(y^m(z) - az\right) dz\right] dp_A$$

The sum of integrated changes will be

$$\begin{aligned}
\int dH(z) &= \sum_{k=0}^{\infty} \int dH^k(z)dz = \sum_{k=0}^{\infty} \left[\int f\left(y^m(z) - az\right) \left[2\mathbf{g}^m(z)\right]^k dz \right] dp_A \\
&= \left[\int f\left(y^m(z) - az\right) \sum_{k=0}^{\infty} \left[2\mathbf{g}^m(z)\right]^k dz \right] dp_A \\
&= \left[\int \frac{1}{1 - 2\mathbf{g}^m(z)} f\left(y^m(z) - az\right) dz \right] dp_A
\end{aligned}$$

Since $dN_A = \int dH(z)$, we find

$$-\frac{dN_A}{dp_A} = \int \frac{1}{1 - 2\mathbf{g}^m(z)} f\left(y^m(z) - az\right) dz$$

which yields (14).

Proof of corollary 5

Consider two values of a, a^l and a^h, $a^l < a^h$. Denote the associated equilibrium distribution, equilibrium mapping, and marginal technological preferences by H^i, Γ^i, and $y_i^m(z)$, respectively, with $i = l, h$. First we want to show that $f(y_h^m(z_i) - a^h z_i) < f(y_l^m(z_i) - a^l z_i)$ for z_i except $z_i = 0$, where they are equal.

First note that $|y_l^m - a^h z_l| > |y_l^m - a_l z_i|$ for $z_i \neq 0$, hence $f(y_l^m - a^h z_l) < f y_l^m - a^l z_i)$ (since $y_l^m(z) < 0$ when $z_i > 0$). It is thus sufficient to show that $|y_h^m(z_i)| > |y_l^m(z_i)|$ for $z_i \neq 0$. To this end, we have that

$$\begin{aligned}
\Gamma^h H^l(z_i) &= F(2 \int g(d(z, z_i)) H^l(z)dz - \mathbf{g} - a^h z_i) \\
&= F(2y_l^m(z_i) - (a^h - a^l)z_i) \\
&> H^l(z_i) \text{ for } z_i < 0
\end{aligned}$$

From lemma 1 it follows that $H^h > \Gamma^h H^l(z_i)$, and hence that $y_h^m(z_i) > y_l^m(z_i)$ for $z_i < 0$. Due to symmetryIt thus follows that $0 > y_l^m(z_i) > y_h^m(z_i)$ for $z_i > 0$, and hence that $|y_l^m(z_i)| < |y_h^m(z_i)|$ for all $z_i \neq 0$ (at $z_i = 0$ $y_l^m(z_i) = y_h^m(z_i) = 0$). Hence $f(y_h^m(z_i) - a^h z_i) < f(y_l^m(z_i) - a^l z_i)$ for all $z_i \neq 0$.

We can now calculate $\mathbf{g}_h^m(z) - \mathbf{g}_l^m(z)$:

$$\mathbf{g}_h^m(z_i) - \mathbf{g}_l^m(z_i) = \int g(d(z, z_i))[f\left(y_h^m(z) - a^h z\right) - f\left(y_l^m(z) - a^l z\right) dz < 0$$

for all $z \neq 0$. It follows that $\int \frac{f^z(y_l^m(z))}{1-2\mathbf{g}_l^m(z)}dz > \int \frac{f^z(y_h^m(z))}{1-2\mathbf{g}_h^m(z)}dz$ and hence that $p_A = p_B$ defined by (15) is increasing in a. This completes the proof.

Proof of Proposition 7

Note that the equilibrium configuration depends on the price difference, $p_A - p_B$, independent of the price level. Hence we can first consider the impact of a higher a on equilibrium configuration, and second address the implications for the price level.

It follows from Lemma 3 that for every $z_i < 0$ then $y^m(z_i)$ increases if a goes up. For $z_i < 0$, the density of the marginal consumer is $f(y^m(z) - az)$. Since $y^m(z) - az > 0$ whenever $z < 0$, and $y^m(z) - az$ increases, it follows that $f(y^m(z) - az)$ declines. For $z > 0$ we have that $y^m(z) - az$ declines. Since $y^m(z) - az < 0$ it follows that $f(y^m(z) - az)$ declines. Thus the set of marginal consumers $\mathbf{g}^m(z)$ declines and equilibrium price increases.

11.5 Proofs related to the section "Endogeneous consumer hetrogeneity"

Proof of Corollary 2

This follows directly by applying equaiton (22) twice.

Compatibility - first order conditions

In any equilibrium, the combination of p_i and θ_i maximizes the profit of firm i given its market share (Armstrong and Vickers 2001). At $z = 0$, all agents have the same number of friends in both networks, hence the indifference curve of a consumer at this point is of the form $\theta_A \frac{\mathbf{g}}{2} - p_A = K$, where K is a constant. Consider a marginal change in p_A and θ_A satisfying the indifference constraint. Yhe new distribution function $\widetilde{H}$ will satisfy $\widetilde{H}(0) = 1/2$, and since $f()$ is symmetric, that $\widetilde{H}(z) = \widetilde{H}(-z)$. Hence the market share of firm A stays constant and equal to $1/2$.

Firm A chooses a combination of compatibility and prices p_A that maximizes profit for a given market share, i.e., solves (assuming symmetry)

$$\max_{\theta_A, p_A} \frac{p_A}{2} - C(\theta_A) \qquad \text{s.t.} \qquad -dp_A + \frac{\mathbf{g}}{2} d\theta_A = 0$$

with first order condition

$$C'(\theta_A) = \mathbf{g}/4$$

as stated in the text.

Two part tariff - first order conditions

We use the same procedure as for compatibility. The indiffference curve of an agent located at $z_i = 0$, with half of his friends in the network, has the form of $-p_A + v(q_A)\mathbf{g}/2 = K$, where K is a constant. Hence

$$\frac{dp_A}{dq_A} = \frac{v'(q_A)\mathbf{g}}{2} = \frac{-x(q_A)\mathbf{g}}{2} \tag{23}$$

Maximizing (20) with respect to q_A subject to (23) yields the first order condition

$$-N_A\frac{x(q_A)\mathbf{g}}{2} + \left[x(q_A) + x'(q_A)(q_A - c)\right]\mathbf{G}_A$$
$$+x(q_A)(q_A - c)\frac{\partial \mathbf{G}_A}{dq_A} = 0$$

or (since $N_A = 1/2$ in the symmetric equilibrium)

$$\frac{1}{2}[1-\gamma]x(q_A) + (q_A - c_x)x'(q_A) + \frac{x(q_A)(q_A - c_x)el_q\mathbf{G}_A}{q_A} = 0$$

where $\gamma := \frac{1}{2}\mathbf{g}/\mathbf{G}_A$ and el_q is the elasticity operator.

Consider a person located at z_i.The derivative of u_A and u_B wrt q_A given (23) gives

$$\frac{du_A(z_i)}{dq_A} = -x(q_A)(\mathbf{g}_A(z_i) - \mathbf{g}/2)$$

Since $\mathbf{g}_A(z_i) > \mathbf{g}/2$ for $z_i < 0$ and $\mathbf{g}_A(z_i) < \mathbf{g}/2$ for $z_i > 0$ it follows that an increase in q_A makes H less concentrated. From Lemma 2 it follows that $\mathbf{G}_A$ decreases. Hence $el_qG_A < 0$.

12 References

Armstrong, M., and Vickers, J. (2001), "Competitive Price Discrimination". *Rand Journal of Economics* **32**, 579-605.

Arthur, W. Brian. (1989), "Competing Technologies, Increasing Returns, and Lock-in by Historical Events." *Economic Journal*, **99**, 116-131.

Banerji, A., and Dutta, B. (2009), "Local network externalities and market segmentation". *International Journal of Industrial Organization* **27**, 605-614.

Bala, V., and Goyal, S. (2000), "A Noncooperative Model of Network Formation". *Econometrica*, **68**, 1181-1229.

Ballester, C., Calvo-Armengol, A., and Zenou, Y. (2006), "Who's Who in Networks. Wanted: The Key Player". *Econometrica*, **74**, 1403-1417.

Birke, D., and Swann, P. (2005), "Network effects and the choice of mobile phone operator". *Journal of Evolutionary Economics* **16**, 65–84.

Cabral, L. (2011), "Dynamic Price Competition with Network Effects", *Review of Economic Studies* 78, 83-111.

Corrocher, N. and Zirulia, L. (2009), "Me and you and everyone we know: An empirical analysis of local network effects in mobile communications " *Telecommunications Policy*, Volume **33**, 68-79.

Farrell, J., and Klemperer, P. (2007), "Coordination and Lock-In: Competition with Switching Costs and Network Effects," In M. Armstrong and R. H. Porter, *Handbook of Industrial Organization*. Amsterdam: Elsevier. 1967-2072 . Farrell, J., and Saloner, G (1985), "Standardization, Compatibility, and Innovation." *Rand Journal of Economics* **16**, 70-83.

—— (1986) "Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation." *American Economic Review* **76**, 940-55.

—— (1992), "Converters, Compatibility, and the Control of Interfaces," *Journal of Industrial Economics* **40**, 9-36.

Fjeldstad, Ø. D. and Sasson, A. (2010) "Membership Matters: On the Value of Being Embedded in Customer Networks", *Journal of Management Studies*, 40, 944-966.

Foros Ø., Hansen B. (2001), "Connecting customers and disconnecting competitors – The facility-based firms' strategy towards virtual operators". *Journal of Network Industries***, 2**, 207-230.

Galeotti, A., and Goyal, S. (2009), "A theory of strategic diffusion". *Rand Journal of Economics*, **40**, 3, 509-532.

Gilbert, RJ (1992), "Introduction to Symposium on Compatibility: Incentives and Market Structure," *Journal of Industrial Economics*, **40**, 1-8.

Granovetter, M (2005), "The Impact of Social Structure on Economic Outcomes". *The Journal of Economic Perspectives* 19, 33-50.

Griva K. and Vettas N. (2011). "Price competition in a differentiated products duopoly under network effects". Information Economics and Policy 23, 85-97

Hoernig S., R Inderst and T Valletti (2011) "Calling Circles: Network Competition with Non-Uniform Calling Patterns", CEIS research paper series Vol. 9, Issue 9, No. 206 – July 2011.

Katz, M. L. and Shapiro, C. (1985) ”Network Externalities, Competition and Compatibility.” *American Economic Review*, **75**, 424-41.

—— (1992) ”Product Introduction with Network Externalities.” *The Journal of Industrial Economics*, **40**, 50-79.

Kranton, R. E. and Minehart, D. F. (2001) ”A Theory of Buyer-Seller Networks.,” *American Economic Review*. **91,** 485-508.

Laffont, J., Rey, P. and Tirole, J. (1998) ”Network Competition: I Overview and Non-Discriminatory Pricing.” *Rand Journal of Economics*, **29**, 1-37.

Rohlfs, J. (1974) ”A Theory of Interdependent Demand for a exchange Service.” *Bell Journal of Economics and Management Science*, **5**, 16-37.

Shy, O. (2001). "The Economics of Network Industries". *Cambridge University Press*, Cambridge MA.

Swann, G.M.P. (2002). "The functional form of network effects". *Information Economics and Policy*, **14**, 417-429.

Spence, M. "Monopoly, Quality and Regulation", *Bell Journal of Economics*, **6**, 417-429.

Stokey, N.L. and Lucas, R.E. (1989), Recursive Methods in Economic Dynamics, *Harvard University Press*.

Sundararajan, A. (2007), Network Effects and Complex Network Structure," *The B.E. Journal of Theoretical Economics*: **7,** (Contributions), Article 46.

Sydsæter, K, A. Strøm and Berck, P. (2005), *Economists' Mathematical Manual*, Springer, Berlin.

Tucker, C. (2008), "Identifying Formal and Informal Influence In Technology Adoption with Network Externalities". *Management Science* **55**, 2024-2039.