Exclusive dealing in desentralized markets

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Prosjektet har mottatt midler fra det alminnelige prisreguleringsfondet.
Abstract

In their influential paper, Aghion and Bolton (1987) argue that a buyer and a seller may agree on high liquidation damages in order to extract rents from future suppliers. As this may distort future trade, it may be socially wasteful.

We argue that Aghion and Bolton’s analysis of entry is incomplete in some respects, as there is only one potential entrant in their model. We construct a model with many potential entrants. Entry is costly, so entering suppliers have to earn a quasi-rent in order to recoup their entry costs. Reducing the entrants’ profits by the help of a breach penalty reduces the probability of entry, and this reduces the attractiveness of breach penalties for the contracting parties.

We show that the initial buyer and seller only have incentives to include a positive breach penalty if there is excessive entry without it, in which case the breach penalty is welfare improving.

Key words: Exclusive contracts, breach penalties, entry, efficiency

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In their influential paper, Aghion and Bolton (1987) argue that a buyer and a seller may have incentives to use partly exclusive contracts in a way that harms welfare. They show

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that liquidation damages awarded to the seller in the event of breach of contract by the buyer (hereafter breach penalties) may be used to extract rents from future suppliers entering the market at a later stage. As a by-product of rent extraction, the most efficient supplier is not always chosen, thus harming economic efficiency. These results have been highly influential and widely applied.  

In this paper we argue that Aghion and Bolton’s’ analysis of entry is incomplete in some respects. First, the model is set in a static (two-stage) setting, while the issue of efficient breach penalties is inherently dynamic. Second, and more importantly, the paper does not model the entry process of new suppliers, or how the supplier and the manufacturer come together in the first place. This is potentially important, as the breach penalty may influence the probability that suppliers show up.

In this paper we study breach penalties in a general equilibrium model with search frictions. We study a search market where suppliers search for manufacturers in an uncoordinated manner (urn-ball process). Heterogenous suppliers may direct their search towards manufacturers without suppliers, or towards manufacturers who already have an (inefficient) supplier. If a manufacturer with a supplier sets a breach penalty, this will reduce the attractiveness of approaching this firm, and hence the probability that a competing supplier shows up is reduced. We show that with Bertrand competition between the suppliers \textit{ex post}, a manufacturer with an inefficient supplier sets the breach penalty to zero, and we conjecture that the equilibrium is efficient. If the returns to entrants exceed the return under Bertrand competition, a manufacturer with an inefficient supplier has an incentive to set a

\footnote{Aghion and Boltons’ findings appear in leading textbooks (Church and Ware 2000, Motta 2004, Pepall, Richards and Norman 2002) as well as in policy analyses. In the ongoing debate surrounding EUs article 82 on dominance, the paper plays a key role. For instance, in a report on Article 82 prepared by the prestigious Economic Advisory Group for Competition Policy in EU (anti-trust group) concerning article 82 (Gual et al 2005), one reads (with explicit reference to Aghion and Bolton): "For example, an exclusive dealing contract that makes entry more difficult may be used to extract rents from a potential entrant." Regarding rebates, the report continues:"Thus, the rebate is analogous to a penalty paid by the entrant; it plays the role of an entry fee, designed to extract some of the efficiency gains of new entrants, and by the same token it creates a barrier to entry".}
strictly positive breach penalty. We show that in the short run (with an exogenous number of suppliers in the market), the socially optimal breach penalty is lower than the equilibrium breach penalty. The reason is that the excessive compensation to the suppliers are corrected for manufacturers with a supplier, but not for manufacturers without a supplier. Hence, with the equilibrium breach penalty, too many suppliers are approaching unmatched suppliers, and too few are approaching matched suppliers. In the long run, with an endogenous number of suppliers, the welfare results are less clear: There will be too much entry of suppliers, and the breach penalty will mitigate this. The socially optimal breach penalty may actually exceed the equilibrium breach penalty.

We also study the effect of less than complete information about breach penalties, by assuming that only a fraction less than one of the suppliers are able to observe the breach penalties set by firms. We show that if sufficiently many (but not all) suppliers observe the breach penalties, the equilibrium is equal to the full-information benchmark. However, if too few suppliers observe the breach penalty, the manufacturing firms will set a strictly positive breach penalty, which is socially inefficient.

A number of studies discuss breach penalties as a remedy for rent extraction, and how this may give rise to an inefficient allocation of resources. A seminal paper (in addition to Aghion and Bolton) is Diamond and Maskin (1977), who analyze breach penalties in a search context (with undirected search). A third important paper is Rasmussen et al (1991) who show that if there are many buyers that cannot coordinate their actions, a seller can bribe some of them to write an exclusive contract and thereby prevent entry (see also Whinston 2000). Fumagalli and Motta (2006) show that naked exclusion cannot be a profitable strategy if the buyers don’t have market power in the market for their final product.

Innes and Sexton (1994) argue that a breach penalty may be warranted if the buyer and the entrant collude against the initial supplier. Marx and Shaffer (1999) consider a retailer monopolist negotiating sequentially with two suppliers. If the initial contract specifies a price below marginal costs, this may affect the bargaining game with the second supplier and enables the monopolist to extract more of the second supplier’s rents. Marx and Shaffer (2007) argue that up-front payments may be used as an exclusion devise in downstream markets.
To our knowledge there are no papers that explicitly model entry in the Aghion-Bolton model. Spier and Whinston (op.cit.) argue that with perfect competition among entrants, the initial buyer and seller have no incentives to set a breach penalty. However, in their model, that is simply because there are no rents to extract from the suppliers. In our model, by contrast, there are rents to extract, but it may not be in the buyer’s and seller’s interest to do so.

1 Model

The model is set in discrete time. There is a fixed set of manufacturers, with mass normalized to 1. The output of each manufacturer is also normalized to 1 per period. Production requires one unit of input from a supplier. The cost for a supplier of serving a manufacturer may be either high, $c_H$, or low, $c_L$. A manufacturer may be in three different states: unmatched, matched with high-cost supplier, or matched with a low-cost supplier. Suppliers enter the market at cost $K$, and their measure is hence endogenous. Between two periods there is an exogenous probability $s$ that a supplier withdraws from the market and receives a continuation value of zero, while the manufacturer becomes unmatched and receives $V^M$ to be defined below. In each period, the timing of the game is as follows:

1. (New suppliers enter the market at cost $K$)

2. Unmatched suppliers (newly entered suppliers, surviving suppliers that were not matched last period and suppliers that were matched last period but lost their contract) draw their type, $\mu$, which is uniformly distributed on the interval $[0, 1]$. The draws are iid over suppliers and time.

3. The suppliers observe whether a manufacturer is unmatched (not in contract with a supplier) or matched (have a supplier), and in the latter case the cost of this supplier and eventually an agreed breach penalty.
4. The unmatched suppliers approach exactly one manufacturer. The suppliers cannot coordinate which of the manufacturer they approach, hence a manufacturer may be approached by zero, one, or many suppliers.

5. The cost of the supplier of delivering to this particular manufacturer is realized. The probability that the cost is low is $\mu$.

6. All the suppliers (if any) that have approached the same manufacturer compete for a contract by submitting bids (to be described above).

7. If the chosen supplier has high costs, the manufacturer and the supplier writes a contract regarding breach of contract. If the supplier is replaced by a low-cost supplier, the supplier is compensated for his loss associated with the breach of contract. In addition, the agents agree on a breach penalty $B$ that the manufacturer has to pay to the incumbent supplier if the contract is cancelled. It follows that $B$ is set so as to maximize joint surplus.

8. Production takes place.

9. Before the next period starts, a fraction $s$ of the suppliers exit the market. The remaining manufacturer enters the next period as unmatched.

**Net present values** Let $V^S(\mu)$ denote the NPV of the expected future income of a supplier of type $\mu$ in the beginning of the period, and let $V^S = E V^S(\mu)$. Let $V^M$ denote the NPV of the expected future income of an unmatched manufacturer. Furthermore, let $M^0$ and $M^1$ denote the NPV of the joint income of manufacturer and a supplier of a high and with a low cost, respectively.

The suppliers that have approached a manufacturer competes for the contract by offering prices. However, it is much more convenient to solve the model in terms of joint incomes and outside options. Let $r$ denote the discount rate and $\beta = 1/(1 + r)$ the discount factor. It follows that the NPV of the joint income of a manufacturer and a supplier with a low cost reads
\[ M^1 = 1 - c_L + \beta [sV^M + (1 - s)M^1] \]

In this expression, \(1 - c_L\) is the joint income in the first period. The second term is the continuation values in the next period, discounted with \(\beta\). With probability \(s\) the match dissolves, the manufacturer receives \(\beta V^M\) and the supplier leaves the market and receives nothing. With probability \((1 - s)\) the match survives, and the NPV joint income is \(M^1\) (the cost of a matched supplier does not change). It follows that

\[ M^1 = [1 - c_L + s\beta V^M] \frac{1 + r}{r + s} \]

Let \(M^{\text{new}}(B)\) denote the joint expected NPV income of the manufacturer and the current supplier if the supplier is replaced by a more efficient one. Note that the breach penalty itself is a transfer between the two agents and hence don’t directly influence joint income. Still \(M^{\text{new}}(B)\) may depend on \(B\), as the breach penalty influences the new supplier’s bid. Let \(q_L^0(B)\) denote the probability that no low-cost supplier emerges next period. It follows that

\[ M^0 = 1 - c_H + \beta \left\{ sV^M + (1 - s) \left[ (1 - q_L^0(B)) M^{\text{new}}(B) + q_L^0(B)M^0 \right] \right\} \]  \hspace{1cm} (1)

Again the first term \(1 - c_H\) denotes the joint income in the current period, and the expression in the square brackets the continuation payoffs. The first term in the square brackets is the continuation value if the match dissolves for exogenous reasons. The first part of the second term is the continuation value of the manufacturer and the supplier if the supplier is replaced. The second part is the continuation value if the match is not destroyed and the supplier is not replaced. \(M^0\) can be rewritten to

\[ M^0 = \left[ 1 - c_H + \beta sV^M + \beta (1 - s)(1 - q_L^0(B)) M^{\text{new}}(B) \right] \frac{1 + r}{1 + r + (1 - s)q_L^0(B)} \]

**Sharing rules**  As a benchmark case we assume that the payoffs to the agents are determined by the Mortensen rule (Mortensen, Kennes). When matched, the suppliers bid for the job, and the manufacturer accepts or rejects the offer. Equivalently, the manufacturer organizes a second price auction without reservation price. Alternative pricing schemes will be discussed below. When making the offer, a high-cost supplier will always propose a breach
penalty that maximizes joint income, i.e., that maximizes $M^0$. We therefore let $M^0$ denote the maximum of $M^0(B)$.

Let $J^S_i$ and $J^M_i$ be payoffs to the supplier and the manufacturer, respectively, where $i = L, H$. Consider first an unmatched manufacturer. The pay-off structure is then as follows:

- Suppose only one supplier shows up. The supplier then obtains the entire match surplus, the manufacturer gets her outside option. Hence the payoffs are
  
  \[ J^S = M^i - \beta V^M \quad (2) \]
  \[ J^M = \beta V^M \]

- Suppose there are more than one supplier. Suppose that there are two or more that have the same (lowest) costs among them. The entire surplus is then allocated to the manufacturer, and we have that
  
  \[ J^S = \beta (1 - s) V^S \quad (3) \]
  \[ J^M = M^i - \beta (1 - s) V^S \quad (4) \]

- With exactly one low-cost supplier and at least one high-cost supplier, the low-cost supplier wins the contract at a price that makes the high-cost supplier indifferent between winning and loosing. Hence
  
  \[ J^S = M^1 - M^0 + \beta (1 - s) V^S \quad (5) \]
  \[ J^M = M^0 - \beta (1 - s) V^S \quad (6) \]

Consider then a matched manufacturer, that already has a high-cost supplier. If the manufacturer replaces the incumbent supplier, it has to compensate her according to the terms of the contract. The value to the manufacturer of continuing the contract is $M^0 - J^S$. If the contract is violated, the manufacturer pays $J^S - \beta (1 - s) V^S + B$ to the old supplier. The manufacturer will thus never accept a bid that provides a utility lower than

\[ M^0 - J^S + (J^S - \beta (1 - s) V^S + B) = M^0 - \beta (1 - s) V^S + B \]
• Suppose exactly one low-cost supplier arrives. The supplier pays the manufacturer her reservation value \( M^0 - \beta(1 - s)V^S + B \), hence

\[
J^S(B) = M^1 - M^0 + \beta(1 - s)V^S - B
\]

provided that the supplier’s outside option does not bind.

• Suppose more than one low-cost supplier arrives. The winning bid makes a low-cost supplier indifferent between winning and losing. The new supplier thus receives his outside option \( \beta(1 - s)V^S \), while the manufacturer before the payment to the incumbent supplier obtains \( M^1 - \beta(1 - s)V^S \). The joint income of the manufacturer and the incumbent supplier is independent of \( B \).

The breach penalty has an effect if exactly one low-cost supplier shows up. In this case, higher breach penalty means higher joint income for the manufacturer and the incumbent supplier (as long as trade occurs). *Ex post*, the new supplier may obtain rents, and the manufacturer and the incumbent supplier can extract these rents by using a breach penalty. In fact, they can expropriate all rent by setting \( B = M^1 - M^0 \). However, as we will see below, the breach penalty reduces the chances that suppliers arrive. Note that the breach penalty does not influence the joint income of the manufacturer and the incumbent supplier in the case where two or more low-cost firms arrive. In this case, the manufacturer extracts all rents from the newcomer in any case.

The payoffs above are in terms of NPV values and joint incomes. However, the results can easily be transferred to prices.

**Search and matching** We consider a symmetric equilibrium. Each supplier has to decide whether to approach an unmatched manufacturer or a manufacturer with an inefficient supplier. Let \( \tau(\mu) \) denote the probability that a supplier of type \( \mu \) approaches an unmatched manufacturer. Since all unmatched manufacturers are identical from the perspectives of the suppliers, they approach each manufacturer with the same probability. Suppliers who approach a matched manufacturer may (and will) let the probability of approaching a given manufacturer depend on the breach penalty required.
We need to introduce some notation regarding various probabilities. For a manufacturer that is not matched initially, let \( p_0, p_1 \) and \( p_2 \equiv 1 - p_1 - p_0 \) denote the probabilities that, respectively, none, exactly one and at least two suppliers arrive, and let \( p_j^L, p_j^H \), \( j = L, H \), denote the corresponding probabilities that low and high cost suppliers arrive.

Let \( v_N, v_L \) and \( v_H \) denote the measure of manufacturers who are unmatched, matched with a low-cost supplier, and matched with a high-cost supplier, respectively. Let \( u \) denote the measure of unmatched suppliers.

The number of suppliers that approach any unmatched manufacturer is thus Poisson distributed with Poisson parameter \( (u/v_N) \int_0^1 \tau(\mu)d\mu \). The probability that a supplier has low cost depends on his type. The number of low-cost suppliers that an unmatched manufacturer receives is Poisson distributed with parameter \( (u/v_N) \int_0^1 \mu \tau(\mu)d\mu \). Hence, in terms of the type-distribution of suppliers that approach a firm, each supplier contributes with a number of "efficiency units" equal to the probability \( \mu \) that it obtains low costs. Analogously, the number of high-cost suppliers is Poisson distributed with parameter \( (u/v_N) \int_0^1 (1 - \mu) \tau(\mu)d\mu \). The probabilities can thus be written as

\[
\begin{align*}
  p_0 &= e^{-(u/v_N) \int_0^1 \tau(\mu)d\mu} \\
  p_0^L &= e^{-(u/v_N) \int_0^1 \mu \tau(\mu)d\mu} \\
  p_0^H &= e^{-(u/v_N) \int_0^1 (1 - \mu) \tau(\mu)d\mu}
\end{align*}
\]

(8)

(9)

By using the sharing rules derived above, it follows that the expected value of searching for an unmatched manufacturer, denoted \( V_N^S(\mu) \), is given by

\[
V_N^S(\mu) = p_0^L p_0^H [\mu(M^1 - \beta V^M) + (1 - \mu)(M^0 - \beta V^M)] + p_0^L (1 - p_0^H) [\mu(M^1 - M^0) + \beta (1 - s) V^S] + (1 - p_0^L) \beta (1 - s) V^S
\]

since \( p_0 = p_0^L p_0^H \) this can be expressed

\[
V_N^S(\mu) = p_0^L \mu(M^1 - M^0) + p_0 [M^0 - \beta (1 - s) V^S - \beta V^M] + \beta (1 - s) V^S
\]

(10)

Consider then a manufacturer with a high-cost supplier (hereafter a matched manufacturer) and a breach penalty \( B \). Corresponding to \( p_0^L \) we define \( q_0^L \) as the probability that no
low-cost supplier arrives, $q_1^L$ the probability that exactly one low-cost supplier arrives, and $q_2^L$ the probability that at least two low-cost suppliers arrive. In equilibrium, all manufacturers that attract suppliers of type $\mu$ must be equally attractive to this type of manufacturer, i.e., the suppliers must receive the same expected value $V^S(\mu)$ when approaching the manufacturers, independently of $B$. The expected income of searching for this manufacturer is

$$V_F^S(\mu; B) = q_0^L(B)\mu(M^1 - M^0 - B) + \beta(1 - s)V^S$$

(11)

$$= V_F^S(\mu)$$

or

$$q_0^L(B)(M^1 - M^0 - B) = \frac{V_F^S(\mu) - \beta(1 - s)V^S}{\mu}$$

(12)

Lemma 1 (Single-crossing). Suppose a supplier of type $\mu^*$ is indifferent between approaching an unmatched and a matched supplier with $B = 0$. Then suppliers of type $\mu > \mu^*$ strictly prefer to search for a matched supplier, while all suppliers of type $\mu < \mu^*$ strictly prefer to search for manufacturers without suppliers.

Proof. It follows from (10) and (11) that

$$\frac{dV_N^S}{d\mu} = p_0^L(M^1 - M^0)$$

$$\frac{\partial V_F^S(\mu; 0)}{\partial \mu} = q_0^L(M^1 - M^0)$$

Since, by definition, $V_N^S(\mu^*) = V_F^S(\mu^*)$, it follows from (10) and (11) that,

$$p_0^L\mu^*(M^1 - M^0) + p_0 [M^0 - \beta V^M - \beta(1 - s)V^S] + \beta(1 - s)V^S$$

$$= \mu^*q_0^L(M^1 - M^0) + \beta(1 - s)V^S$$

Hence

$$p_0^L\mu^*(M^1 - M^0) < \mu^*q_0^L(M^1 - M^0)$$

which implies

$$\frac{dV_N^S}{d\mu} < \frac{\partial V_F^S(\mu; 0)}{\partial \mu}$$

The result thus follows. ■
If a supplier approaches an unmatched manufacturer, it may get a positive surplus even if it obtains high costs, if no other suppliers emerge. If it approaches a matched manufacturer, it only receives a positive surplus if it obtains low costs (and is the only one with low cost). It follows that approaching a matched manufacturer is relatively more attractive the higher is \( \mu \), and that if any suppliers prefer to approach a matched manufacturer, it must be those with a higher \( \mu \).

An important observation is that (12) with \( B = 0 \) inserted must hold for all \( \mu \geq \mu^* \). Hence the right-hand side of the equation must be independent of type \( \mu \). In particular this implies that for any \( \mu \geq \mu^* \) we have that

\[
\frac{V_F^S(\mu) - \beta(1-s)V^S}{\mu} = \frac{V_F^S(\mu^*) - \beta(1-s)V^S}{\mu^*}
\]

or that (recall that \( V_F^S(\mu^*) = V_N^S(\mu^*) \))

\[
V_F^S(\mu) = \frac{\mu}{\mu^*}V_N^S(\mu^*) + (1 - \frac{\mu}{\mu^*})\beta(1-s)V^S\tag{14}
\]

Another interesting observation is the following. Suppose \( \mu^* < 1 \), and consider a manufacturer with a breach penalty. Consider a supplier with \( \mu = 1 \). From (12) it follows that in equilibrium the supplier must be get its equilibrium pay-off if approaching this firm, hence we must have that

\[
q_L^*(B)(M^1 - M^0 - B) = V^S(1) - \beta(1-s)V^S
\]

This equation uniquely determines \( q_L^*(B) \) up to the point where \( q_L^*(B) \geq 1 \), in which case the manufacturer does not attract any suppliers. Since the right-hand side of (12) is independent of \( \mu \), it follows that \( q_L^*(B) \) satisfies (12) for all \( \mu \). This is intuitive, as \( \mu \) is thus a multiplicative factor in (11) and hence does not influence the trade-off between \( q_L^* \) and \( B \). In particular, the next lemma follows immediately

**Lemma 2** The cut-off \( \mu^* \) is independent of \( B \).

For any \( B \), \( q_L^* \) is implicitly defined by (12). The number of low-cost suppliers that arrives is again Poisson distributed, with a generic Poisson parameter that we generically denote by \( x \) and which will depend on \( B \). Hence

\[
q_L^* = e^{-x}\tag{15}
\]
From (12), (13) and the definition of \( \mu^* \), \( V_N^S(\mu^*) = V_F^S(\mu^*) \), it follows that \( x \) is defined by

\[
e^{-x}(M^1 - M^0 - B) = \frac{V_N^S(\mu^*) - \beta(1 - s)V^S}{\mu^*} \tag{16}\]

which defines \( x = x(B) \). It follows that

\[
q_0^L(B) = e^{-x(B)} = \frac{V_N^S(\mu^*) - \beta(1 - s)V^S}{(M^1 - M^0 - B) \mu^*} \tag{17}\]

For a given \( V_N^S(\mu^*) \), this equation uniquely determines \( x(B) \).

**Expected values**  Let \( V^S(\mu) = \max[V_N^S(\mu), V_F^S(\mu)] \). Recall that \( V^S = EV^S(\mu) \). It follows from (10) and (14)

\[
V^S = p_0^L \mu^* x^2 (M^1 - M^0) + p_0 \mu^* [M^0 - \beta V^M - \beta(1 - s)V^S] + q_0^L(B) \frac{1 - \mu^* x^2}{2} (M^1 - M^0 - B) + \beta(1 - s)V^S \tag{18}
\]

The expected income of an unmatched manufacturer, \( V^M \), is given by the following expression

\[
V^M = p_2 \left(M^0 - \beta(1 - s)V^S - \beta V^M\right) + p_2^L (M^1 - M^0) + \beta V^M \tag{19}
\]

Finally, the joint income of the manufacturer and the incumbent supplier if a new supplier shows up (i.e. conditional on the event that at least one low-cost supplier shows up), is given by

\[
M^{new}(B) = \frac{q_1^L}{1 - q_0^L} (M_0 + B) + \left(1 - \frac{q_1^L}{1 - q_0^L}\right) M_1 \tag{20}
\]

**Aggregate consistency**  Recall that \( v_N \) is the measure of unmatched manufacturers. Inflow equal to outflow implies that

\[
(1 - p_0)v_N = s(1 - v_N) \tag{22}\]

Inflow equal to outflow to \( v_H \) (manufacturers with high-cost suppliers), provided that all firms set the same \( B \), reads

\[
p_0^L (1 - p_0^H) v_N = (p_0^L - p_0^H)v_n \tag{23}
\]

\[
= (1 - q_0^L(B) + s)v_H \tag{24}
\]
The left-hand side of the equation shows the inflow until \( v_H \), the stock of badly matched firms. The right-hand side shows the outflow.

For completeness; inflow equal to outflow for \( v_L \) (which follows from (22) and (23))

\[
(1 - p_0^L) v_N + (1 - q_0^L(B)) v_H = s v_L
\]

Finally, \( u \) is determined by a free entry condition (see below).

The number of efficiency units of suppliers approaching matched manufacturers is given by \( u \int_1^1 d \mu = u(1 - \mu^*)/2 \), hence

\[
x(B) = \frac{u(1 - \mu^*)}{2v_H}
\]

## 2 Equilibrium

We are now ready to define our equilibrium

**Definition 3** An equilibrium of the model is a breach penalty \( B^* \), a cut-off \( \mu^* \), a measure \( u^* \)
of unmatched suppliers, measures \( v_N^* \) and \( v_H^* \) of unmatched manufacturers and manufacturers
matched with a high-cost suppliers, such that

1. Optimal cut-off: \( V_N^S(\mu^*) = V_F^S(\mu^*) \)
2. Optimal breach penalty: \( B \) maximizes \( M^0 \) s.t. (17)
3. Optimal entry: \( EV(\mu) = K \)
4. Aggregate consistency: The flow equations (22)-(23) are satisfied

**Conjecture 4** The equilibrium exists and is unique

We now want to derive the optimal breach penalty. By inserting (21) into (1) it follows that

\[
M^0 = 1 - c_H + \beta \left[ s V^M + (1 - s)(q_1^L(B)B + q_2^L(B)(M_1 - M_0) + M_0) \right]  
\]
Define the joint per period gain from search for the manufacturer and the incumbent supplier, $m_0$, as

$$m_0 = q^L_1(B)B + q^L_2(B)(M_1 - M_0) + M_0$$

The agents set $B$ so as to maximize the gain from search. Recall that the number of low-cost suppliers that approaches a matched firm is Poisson distributed with parameter $x$. It is a property of the Poisson distribution that

$$\frac{dq^L_1}{dx} = q^L_0 - q^L_1$$
$$\frac{dq^L_2}{dx} = q^L_1$$

Hence

$$\frac{dm_0}{dB} = q^L_1 + [(q^L_0 - q^L_1)B + q^L_1(M_1 - M_0)]\frac{dx}{dB} + (1 - q^L_2)\frac{dM_0}{dB} \quad (26)$$

From (16) we get that

$$\frac{dx}{dB} = -\frac{1}{M^1 - M^0 - B}$$

which inserted gives

$$\frac{dm_0}{dB} = -\frac{q^L_1B}{M^1 - M^0 - B} - (1 - q^L_2)\frac{dM_0}{dB}$$

We have that $\frac{dM_0}{dB}$ is zero if and only if $\frac{dm_0}{dB}$ is zero. We have thus shown the following result

**Proposition 5** The equilibrium breach penalty $B^*$ is equal to zero

### 3 Alternative price mechanisms

In this section we study other price setting mechanisms than Betrand competition, which allocates a larger share of the surplus to the supplier. This may give the firms an incentive to set a breach penalty greater than zero. We assume that a supplier, in the event that it faces one or more low cost competitors, receives an expected pay-off of $\Delta(x)$, which may depend on the entry intensity, $x$. It follows that (11) can be written as (for $\mu = \mu^*$)

$$q^L_0(B)(M^1 - M^0 - B) + (1 - q^L_0(B))\Delta(x) = \frac{V^{S}(\mu^*) - \beta(1 - s)V^{S}}{\mu^*}$$
It follows from (16) that
\[ e^{-x}(M^1 - M^0 - B) + (1 - e^{-x}) \Delta(x) = \text{Const} \]
and hence that
\[ \frac{dx}{dB} = -\frac{1}{M^1 - M^0 - B - \Delta(x) + (1 - e^{-x}) \Delta'(x)} \] (27)
The expected "gain from search" is
\[ m_0 = q^L_1(B)B + q^L_2(B)(M_1 - M_0) + M_0 - (1 - q^L_0)x\Delta(x) \]
The last term captures the expected cost associated with the transfer to suppliers. If more than one low cost firm approach the manufacturer, the ex ante cost is the per entrant transfer \( \Delta(x) \) multiplied with the expected number of entrants.\(^2\)

Analogous to (26), we get that
\[ \frac{dm_0}{dB} = \frac{dM_0}{dB} + \left[ q^L_0(B)B + q^L_2(B)(M_1 - M_0) \right] \frac{dx}{dB} \]
\[ + (1 - q^L_2) \frac{dM_0}{dB} + \left[ q^L_1(B)(B + \Delta(x)) + q^L_1(M_1 - M_0) \right] \frac{dx}{dB} \]
\[ = -\frac{q^L_0 B}{M^1 - M^0 - B - \Delta} - (1 - q^L_0) \frac{dM_0}{dB} \]
Again we have that \( \frac{dM_0}{dB} \) is zero if and only if \( \frac{dm_0}{dB} \) is zero. Hence the first-order conditions form maximum reads
\[ q^L_0 B = (1 - q^L_0) \Delta(x) \]
\(^2\)The term can be explained as follows: denote by \( \rho_i \) the probability that \( i \) low cost suppliers arrive, thus \( q^L_2 = \rho_2 + \rho_3 + \ldots \). Then
\[ m_0 = q^L_1(B)B + q^L_2(B)(M_1 - M_0) - \Delta [2\rho_2 + 3\rho_3 + \ldots] \]
The last term can be expressed
\[ [2\rho_2 + 3\rho_3 + \ldots] = e^{-x} \left[ \frac{x^2}{2!} + 3\frac{x^3}{3!} + \ldots \right] \]
\[ = xe^{-x} \left[ \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \right] \]
\[ = x(1 - q^L_0) \]
Or

\[ B = \frac{1 - q_0^L}{q_0^L} \Delta(x) > 0 \]  

(28)

**Proposition 6**  If the suppliers are overcompensated as described above, the matched manufacturers set a strictly positive breach penalty

The next question is whether the breach penalty reduces welfare. Suppose first that the suppliers are not over-compensated if they search for an unmatched supplier. In this situation we have the following result

**Conjecture 7**  Suppose suppliers searching for a matched manufacturer are overcompensated as described above, while they are not overcompensated if they search for unmatched manufacturers. Then the equilibrium breach penalty is socially optimal

**Proof.**  The expected surplus of searching for a matched supplier is

\[ V^S(\mu; B) = q_0^L \mu \left( M^1 - M^0 - B \right) + (1 - q_0^L) \mu \Delta(x) + \beta(1 - s) V^S \]

inserting for \( B \) from (28)

\[ V^S(\mu; B) = q_0^L \mu \left( M^1 - M^0 - \frac{1 - q_0^L}{q_0^L} \Delta(x) \right) + (1 - q_0^L) \mu \Delta(x) + \beta(1 - s) V^S = q_0^L \mu \left( M^1 - M^0 \right) + \beta(1 - s) V^S \]

thus \( B \) cancels out the expected transfer. □

Since the optimal breach penalty corresponds to the ex ante expected surplus, the Bertrand equilibrium, \( \Delta = 0 \), is replicated.

## 4 Price-insensitive suppliers

A crucial assumption in the analysis so far is that the suppliers, when approaching a matched manufacturer, knows her breach penalty. In this section we alter this assumption. We assume that if a supplier decides to search for a matched manufacturer, there is a probability \( \kappa \) that it will not observe (any of the) breach penalties \( B \).
Uninformed suppliers searching for a matched manufacturer randomizes between all matched manufacturers. Thus a manufacturer expropriating all surplus by setting $B$ equal to $M^1 - M^0$ may still get applicants. Informed suppliers optimize given the observed $B$ distribution.

A sufficiently small $\kappa$ replicates the equilibrium with informed suppliers only. All firms set $B = 0$ and there are no gains associated with being informed. This equilibrium holds as long as $\kappa$ is so low that no firm has an incentive to deviate, that is set $B = M^1 - M^0$ and expropriate the surplus from uninformed suppliers. A firm that deviates (single shot) obtains:

$$M^{0D} = 1 - c_H + \beta \left[ sV^M + (1 - s) \left( (1 - q^L_0) \kappa(M_1 - M_0) + M^0 \right) \right]$$

where $q^L_0$ follows from (15), whereas the equilibrium profit is

$$M^0 = 1 - c_H + \beta \left[ sV^M + (1 - s) \left( (1 - q^L_0 - q^L_1)(M_1 - M_0) + M_0 \right) \right]$$

The all informed equilibrium is replicated if

$$(1 - q^L_0) \kappa \leq (1 - q^L_0 - q^L_1)$$

**Proposition 8** There is a threshold value $\kappa^{\text{min}} > 0$ so that the equilibrium replicates the full information equilibrium for all $\kappa < \kappa^{\text{min}}$

If $\kappa$ exceeds $(1 - q^L_0 - q^L_1)/(1 - q^L_0)$ some manufacturers set $B = M^1 - M^0$ in equilibrium, and these firms are approached by uninformed suppliers only. All remaining manufacturers set $B = 0$ and are attracted by any supplier searching for a matched firm.

Note that an matched firm will set $B$ either equal to 0 or equal to $M^1 - M^0$. The intuition is as follows: as long as a positive measure of informed suppliers approaches the firm, the marginal impact on the arrival rate from increasing $B$ corresponds to the impact a higher $B$ has in the symmetric information version of the model. However there will be a kink in aggregate response at the value of $B$ at which it is only uninformed suppliers that approach the firm. Above this level, expected profit is strictly increasing in $B$ up to the level at which the supplier withdraw his offer.
Suppose now that \( \kappa > \kappa^{\text{max}} \). Suppose further that at least some suppliers search for matched employees. Denote by \( q_{0B}^L \) the probability that no low cost firm approaches a manufacturer with positive \( B \).

In equilibrium all matched firms are indifferent between the two alternatives, which yields the condition

\[
1 - q_{0B}^L = 1 - q_0^L - q_1^L
\]

Let \( v_H \) denote the measure of firms matched with a high cost supplier, and denote by \( v_0^H \) the subset of badly matched firms that set \( B = 0 \). Informed suppliers searching for a matched firm will choose a firm with \( B = 0 \), whereas uninformed suppliers randomize. A firm choosing \( B = 0 \) is approached by low cost suppliers with intensity

\[
\lambda_H : = \left[ (1 - \kappa) \frac{u}{v_0^H} + \kappa \frac{u}{v_H} \right] \left( \frac{1 - \mu_s^2}{2} \right)
\]

\[
= \frac{u}{v_0^H} \left[ 1 - \kappa \left( 1 - \frac{v_0^H}{v_H} \right) \right] \left( \frac{1 - \mu_s^2}{2} \right)
\]

If the firm sets \( B = M^1 - M^0 \) only uninformed suppliers arrive. Hence low cost suppliers arrive with intensity

\[
\lambda_H^B = \kappa \frac{u}{v_H} \left( \frac{1 - \mu_s^2}{2} \right)
\]

Conjecture 9 There exists a \( \kappa^{\text{max}} < 1 \) such that for any \( \kappa > \kappa^{\text{max}} \), no suppliers search for matched employers

Proof. Suppose not. Let \( q^* \) be defined as

\[
q^*(M_1 - M_0) = V_S^S(1 - \beta(1 - s)V_S^S
\]

That is, \( q^* \) is the lowest probability of a "fruitful" match in the matched-manufacturer market that makes the best type is willing to enter this market. Hence for a market to exist we must have that

\[
\kappa \frac{v_0^H}{v_H} q_0^L + (1 - \kappa) q_0^L \geq q^*
\]

Since \( q_0^L \leq 1 \), it follows that

\[
\frac{v_0^H}{v_H} \geq \frac{q^* - (1 - \kappa)}{\kappa}
\]
which goes to \( q^* \) as \( \kappa \to 1 \). Furthermore, as \( v_H \) does not go to zero as \( \kappa \) goes to 1, it follows that \( \lim_{\kappa \to 1} v_H^0 > 0 \). From (30) and (31) it follows that

\[
\lim_{\kappa \to 1} \frac{\lambda_H}{u} = \frac{\lambda_B^B}{u}
\]

If \( \mu^* \) if strictly below one as \( \kappa \to 1 \), then

\[
\lim_{\kappa \to 1} 1 - q_{0B}^L < \lim_{\kappa \to 1} 1 - q_0^L - q_1^L
\]

If \( \mu^* \) converges to one as \( \kappa \to 1 \), then \( q_{0B}^L \to 1 \) and \( q_0^L \to 1 \). Then it follows that the probability that exactly one firm enters, conditional on at least one firm entering, converges to one. Denote by

\[
v = \frac{u}{v_H} \left( \frac{1 - \mu^*^2}{2} \right)
\]

then

\[
\lim_{\kappa \to 1} \frac{q_{1B}^L}{1 - q_{0B}^L} = \lim_{\kappa \to 1} \frac{q_1^L}{1 - q_0^L} = \frac{ve^{-v}}{1 - e^{-v}}
\]

In the limit, as \( v \) converges to 0, it follows from L’Hôpital’s rule that

\[
\frac{ve^{-v}}{1 - e^{-v}} \to 1
\]

Hence no manufacturing firm will set \( B = 0 \), and we have derived a contradiction. ■

A supplier searching for a matched firm obtains

\[
V_F^S(\mu|\mu^*) = \left[ 1 - \kappa \left( 1 - \frac{v_H^0}{v_H} \right) \right] q_0^L \mu(M^1 - M^0) + \beta(1 - s)\nu_S
\]

where \( v_H^0/v_H \) is the proportion of matched firms that set \( B = 0 \).

In equilibrium

\[
V_F^S(\mu^*|\mu^*) = V_N^S(\mu^*|\mu^*)
\]

The expected supplier value can be expressed (see appendix for details)

\[
V^S = \frac{v_N}{u} \left[ p_1^L (M^1 - M^0) + p_1 (M^0 - \beta V^M - \beta(1 - s)\nu^S) \right] + \frac{v_H^0}{u} q_1^L (M^1 - M^0) + \beta(1 - s)\nu_S
\]

which corresponds to (18) in the full information case.

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5 Concluding remarks

We have discussed the extent to which an incumbent buyer and seller have incentives to extract rents from entering suppliers by using a breach penalty. We argue that as long as the entrants obtain zero profits \textit{ex ante}, and there is Bertrand competition between the suppliers \textit{ex post}, the optimal breach penalty is zero. A positive (negative) breach penalty will only be profitable to the incumbents if the entrants have too strong (weak) incentives to enter. Even in this case, the breach penalty that maximizes profits is constrained efficient.

We conjecture that rent extraction in general is less attractive when entry is taken into account, and that the social and the private incentives to extract rents from the entrants generally coincide as long as the entrants obtain zero profits. For instance, Bernheim and Whinston (1998) model a more complex environment, where one buyer and \textit{two} sellers are present at the contracting stage. Later on, a new \textit{buyer} may arrive. Bernheim and Whinston show that the initial agents’ joint profit may be maximized if one of the sellers is excluded from the market, as this will reduce the competition for delivery to the entering buyer. With endogenous entry of new buyers, such rent extraction will reduce the probability of entry. We conjecture that when the incumbent buyer and seller take entry into account, the incentive to exclude one of the sellers will be eliminated.

6 Appendix

Proof of (18)
This expression follows directly from (10) and (11) and the definitions of $p^L_1$, $p_1$ and $q^L_1$:

$$V^S = \int_0^{\mu^*} V^N d\mu + \int_{\mu^*}^1 V^f d\mu$$

$$= \int_0^{\mu^*} \left[ p^L_0 \mu(M^1 - M^0) + p_0 \left[ M^0 - \beta V^M - \beta(1 - s)V^S \right] + \beta V^S \right] d\mu$$

$$+ \int_{\mu^*}^1 \left[ q^L_0 (B) \mu(M^1 - M^0 - B) + \beta V^S \right] d\mu$$

$$= \frac{p^L_0 \mu^2}{2} (M^1 - M^0) + p_0 \mu^* \left[ M^0 - \beta V^M - \beta(1 - s)V^S \right] + \mu^* \beta(1 - s)V^S$$

$$+ q^L_0 (B) \frac{1 - \mu^2}{2} (M^1 - M^0 - B) + (1 - \mu^*) \beta(1 - s)V^S$$

$$= \frac{V^N}{u} \left[ p^L_1 (M^1 - M^0) + p_1 \left[ M^0 - \beta V^M - \beta(1 - s)V^S \right] \right]$$

$$+ \frac{V^H}{u} q^L_1 (B) (M^1 - M^0 - B) + \beta(1 - s)V^S$$

**Sign of $dM_0/dB$**

We want to prove that the sign of $\frac{dM_0}{dB}$ is the same as the sign of $\frac{dm_0}{dB}$. From (25) it follows that

$$M_0 = 1 - \tau + \beta \left[ sV^M + (1 - s)(m_0 + M_0) \right]$$

Which gives

$$\frac{dM_0}{dB} = \beta(1 - s) \left( \frac{dm_0}{dB} + \frac{dM_0}{dB} \right)$$

or

$$\frac{dM_0}{dB} (1 - \beta(1 - s)) = \frac{dm_0}{dB}$$

The result thus follows.

**Uniqueness**

Assume there is more than one equilibrium, referred to as $\mu^*_a$ and $\mu^*_b$, with $\mu^*_a < \mu^*_b$. For reference we have

$$V^S(\mu|\mu^*) = e^{-\theta_\mu \left( \frac{1 - \mu^2}{2} \right)} \mu(M^1 - M^0) + \beta(1 - s)V^S$$

(33)

$$V^S_N(\mu|\mu^*) = e^{-\theta_\mu \frac{\mu^2}{2}} \mu(M^1 - M^0) + e^{-\theta_\mu \mu^*} \left[ M^0 - \beta V^M - \beta(1 - s)V^S \right] + \beta(1 - s)V^S$$

$$V^M = (1 - \theta_\mu \mu^* e^{-\theta_\mu \mu^*} - e^{-\theta_\mu \mu^*}) \left( M^0 - \beta (1 - s)V^S - \beta V^M \right)$$

(34)

$$+ \left( 1 - \theta_\mu \frac{\mu^2}{2} e^{-\theta_\mu \frac{\mu^2}{2}} - e^{-\theta_\mu \frac{\mu^2}{2}} \right) (M^1 - M^0) + \beta V^M$$

(35)
Refer to $V^{S}(\mu|\mu^{*}) = V^{S}(\mu|\mu^{*}) - \beta(1-s)V^{S}$ and $v_{N}(\mu|\mu^{*}) = v_{N}(\mu|\mu^{*}) - \beta(1-s)V^{S}$:

$$V^{S}(\mu|\mu^{*}) = e^{-\theta\mu^{(1-\mu^{*})^{2}/2}}\mu(M^{1} - M^{0})$$

$$v_{N}(\mu|\mu^{*}) = e^{-\theta_{a}\mu^{*^{2}/2}}\mu(M^{1} - M^{0}) + e^{-\theta_{a}\mu^{*}}\left[M^{0} - \beta V^{M} - \beta(1-s)V^{S}\right]$$

Obviously, for all $\mu$

$$V^{S}(\mu|\mu_{a}^{*}) < V^{S}(\mu|\mu_{b}^{*})$$

thus for both $\mu_{a}^{*}$ and $\mu_{b}^{*}$ to be equilibria we must have

$$v_{N}(\mu|\mu_{a}^{*}) < v_{N}(\mu|\mu_{b}^{*})$$

which requires that $\beta V^{M} + \beta(1-s)V^{S}$ declines.

$$V^{S} = \int_{0}^{\mu^{*}} \left(p_{0}^{L}\mu(M^{1} - M^{0}) + p_{0}[M^{0} - \beta V^{M} - \beta(1-s)V^{S}]\right)$$

$$+ \int_{\mu^{*}}^{1} \left(q_{0}^{L}(B)\mu(M^{1} - M^{0} - B)\right) + \beta(1-s)V^{S}$$

$$= \frac{V^{S}}{u} \left[p_{1}^{L}(M^{1} - M^{0}) + p_{1}[M^{0} - \beta V^{M} - \beta(1-s)V^{S}]\right]$$

$$+ \frac{V^{S}}{u} q_{1}^{L}(B)(M^{1} - M^{0} - B) + \beta(1-s)V^{S}$$

The expected income of an unmatched manufacturer, $V^{M}$, is given by the following expression

$$\begin{align*}
V^{M} &= p_{2}(M^{0} - \beta(1-s)V^{S} - \beta V^{M}) + p_{2}^{L}(M^{1} - M^{0}) + \beta V^{M} \\
&= (36)
\end{align*}$$

From (34) it follows that $V^{M}$ must increase, thus $V^{S}$ must decline. Since $V^{S}$ can be expressed

$$V^{S} = \int_{0}^{\mu^{*}} \left(e^{-\theta\mu^{*^{2}/2}}\mu(M^{1} - M^{0}) + e^{-\theta\mu^{*}}\left[M^{0} - \beta V^{M} - \beta(1-s)V^{S}\right]\right)$$

$$+ \int_{\mu^{*}}^{1} e^{-\theta(1-\mu^{*^{2}/2})}\mu(M^{1} - M^{0}) + \beta(1-s)V^{S}$$

$$V^{S} = \frac{1}{1 - \beta(1-s)} \int_{0}^{\mu^{*}} e^{-\theta\mu^{*^{2}/2}}\mu(M^{1} - M^{0})$$

$$+ e^{-\theta\mu^{*}}\left[M^{0} - \beta V^{M} - \beta(1-s)V^{S}\right] + \int_{\mu^{*}}^{1} e^{-\theta(1-\mu^{*^{2}/2})}\mu(M^{1} - M^{0})$$

it follows that $V^{S}$ must increase which yields a contradiction.
Derivation of VS

\[
V^S - \beta (1-s) V^S = (1 - \kappa) \left[ \int_0^{\mu_U} (p_0^L \mu (M^1 - M^0) + p_0 [M^0 - \beta V^M - \beta (1-s) V^S]) + \int_{\mu_L}^{1} (q_0^L \mu (M^1 - M^0)) \right] + \kappa \left[ \int_0^{\mu_U} (p_0^L \mu (M^1 - M^0) + p_0 [M^0 - \beta V^M - \beta (1-s) V^S]) + \int_{\mu_L}^{1} \left( \frac{v_H^0}{v_H} q_0^L \mu (M^1 - M^0) \right) \right]
\]

\[
= (1 - \kappa) \left[ \int_0^{\mu_U} \left( p_0^L \mu \frac{\mu_2^2}{2} (M^1 - M^0) + p_0 \mu [M^0 - \beta V^M - \beta (1-s) V^S] \right) + \int_{\mu_L}^{1} (q_0^L \mu (M^1 - M^0)) \right] + \kappa \left[ \int_0^{\mu_U} \left( p_0^L \mu \frac{\mu_2^2}{2} (M^1 - M^0) + p_0 \mu [M^0 - \beta V^M - \beta (1-s) V^S] \right) + \int_{\mu_L}^{1} \left( \frac{v_H^0}{v_H} q_0^L \mu \frac{\mu_2^2}{2} (M^1 - M^0) \right) \right]
\]

\[
= \left[ (1 - \kappa) \left( p_0^L \mu \frac{\mu_2^2}{2} (M^1 - M^0) + p_0 \mu [M^0 - \beta V^M - \beta (1-s) V^S] \right) \right] + \kappa \left[ (1 - \kappa) q_0^L \mu (M^1 - M^0) - (1 - \kappa) q_0^L \frac{\mu_2^2}{2} (M^1 - M^0) \right]
\]

More on the imperfect information case

Note that \( v_H \), the set of bad matched firms in (18), is in (32) replaced by \( v_H^0 \), the subset without breach penalty. Intuitively matched firms with breach penalty expropriates all surplus and thus do not contribute to the expected surplus of unmatched suppliers.

\[
V^M = p_2 \left( M^0 - \beta (1-s) V^S - \beta V^M \right) + p_2^L (M^1 - M^0) + \beta V^M
\]

Inflow equal to outflow for non-matched firms implies that

\[
(1 - p_0) v_N = s(1 - v_N)
\] (37)

where \( p_0 \) is the probability that no low cost firm arrives.
Inflow equal to outflow to $v_H$ (high cost),

\[
(p^L_0 - p_0) v_N = (1 - q^L_{0B} + s) (v_H - v^0_H) + (1 - q^L_0 + s)v^0_H
\]  

Finally, inflow equal to outflow for $v_L$ (which follows from the other two conditions)

\[
(1 - p^L_0) v_N + (1 - q^L_{0B}) (v_H - v^0_H) + (1 - q^L_0)v^0_H = s v_L
\]

Finally, suppliers approach unmatched manufacturers with intensity

\[
\lambda_N = \frac{u}{v_N} \mu^*\]

thus

\[
p_0 = e^{-\lambda_N}
\]

whereas low cost suppliers arrive with intensity

\[
\lambda^L_N = \frac{u}{v_N} \frac{\mu^* 2}{2}
\]

which yields

\[
p^L_0 = e^{-\lambda^L_N}
\]

The characterization of the equilibrium follow to the full information case.

An increase in $\kappa$ has an immediate impact on $V^f$ (given $v_H$, $v_L$, $v_n$ and $u$). First we can show that $V^S_F$ shifts downwards. Assume not. Then $\mu^*$ decreases and thus $q^L_{0B}$ decreases. Then it follows from (29) that $q_0$ decreases, which yields a contradiction. Thus $V^S_F$ shifts downwards and $\mu^*$ increases.

A higher $\mu^*$ impacts $V_n$ through $p_0$ and $p^L_0$ which both decline.

We have two conditions: Manufacturers with bad match indifferent between high and zero $B$ requires that

\[
q^L_{0B} = q^L_0 + q^L_1
\]

Supplier indifferent, that is $\mu^*$:

\[
\left[1 - \kappa \left(1 - \frac{v^0_H}{v_H}\right)\right] q^L_0 \mu^* (M^1 - M^0) = p^L_0 \mu^* (M^1 - M^0) + p_0 [M^0 - \beta V^M - \beta (1 - s) V^S]
\]

Single-crossing:

\[
\left[1 - \kappa \left(1 - \frac{v^0_H}{v_H}\right)\right] q^L_0 > p^L_0
\]
The probabilities are:

\[ q_0^L = e^{-\frac{u}{v_H} \left[ 1 - \kappa \left( 1 - \frac{v_H}{v_H} \right) \right] \left( \frac{1 - \mu^2}{2} \right) } \]
\[ q_{\alpha B}^L = e^{-\frac{\kappa u}{v_H} \left( \frac{1 - \mu^2}{2} \right) } \]
\[ p_0^L = e^{-\frac{u}{v_N} \frac{\mu^2}{2} } \]

References


Church, J and Ware, R (2000), Industrial Organization, McGraw-Hill.


