Upward Pricing Pressure and structural market changes
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Prosjektet har mottatt midler fra det alminnelige prisreguleringsfondet.
Abstract

If one firm buys inputs from a competitor, the input price may be used to internalize the competition between the firms. Thus, positive pricing pressure may arise if one firm starts to buy inputs from a competitor. Conversely, pricing pressure may be significantly reduced if two firms with vertical restraints merge, as pre-merger competition already was internalised through the input price. We present a method for adjusting the formula of Hausman et al. (2011), in order to predict correct pricing pressures not only for horizontal mergers, but also for other structural market changes.

Short phrases: Merger analysis, Upward Pricing Pressure, Vertical restrictions

JEL Classification: L44, L42

1 Introduction

The upward pricing pressure (UPP) is a measure of one-sided incentives for a firm to raise its price following a horizontal merger, developed by Farrell and Shapiro (2010) and Werden (1996). The measure is based on pre-merger margins, sale quantities, and diversion ratios. Inasmuch as it requires substantially less data and resources than a full merger simulation, it has quickly become an important tool for competition authorities, when assessing the effects of mergers. The European Commission has used this type of analysis in several telecom cases; Hutchinson 3G Austria /Orange (M.6497), Hutchinson 3G UK/Telefonica Irland (M.6992) and Telefonica Deutschland/ Eplus (M.7018).

The central idea of the UPP analysis developed by Farrell and Shapiro (2010) is that a merger will remove the competitive constraint that the parties exerted on each other prior to the merger. Based on this idea, several different indicators have been developed and used in practice, see for instance Oledale and Padilla (2013). Furthermore, the method has been extended by Wilig (2011), to account for changes in product quality and partial equity stake in a competing firm.

Most UPP indicators are just that, indicators of pricing pressure. Hausman et al. (2011) have extended the UPP method, by developing a formula for calculating the pricing pressure following a merger. This is achieved by identifying the post-merger first-order-conditions with variables that can be observed prior to the merger. This formula was used by the European Commission in
the Austrian telecom case (M. 6497) and by the Norwegian Competition Authority in the Tele2/TeliaSonera merger case in 2013.

The test, however, does not recognize that competition may be partly internalized pre-merger, for instance due to vertical relations. Nor does the test explicitly include the possibility of changing marginal costs, for instance due to merger-related efficiency gains. Furthermore, the formula can only be used to calculate the pricing pressure following a merger, and not other types of structural changes.

In reality, vertical relations are not uncommon between downstream competitors. Such vertical relations need to be taken into account, as they may affect the firms’ incentives pre- and post-merger. This can be illustrated by an example. Assume that two firms (A and B) are competing downstream, and that A buys inputs from B. If the input price charged by firm B is higher than its marginal cost, both firms will charge higher prices - compared to the situation where A is able to buy inputs at a price equal to B’s marginal cost. This is due to firm A having a larger marginal cost (causing A to set a higher pre-merger price) and firm B having a margin on A’s sales (inducing B to compete less fiercely for A’s customers). Thus, competition is, at least partly, internalized through the input price.

Conducting a UPP-test on a merger between the two firms in the example, based on the framework developed by Hausman et al. (2011), will predict incorrect pricing pressure. This is caused by the test implicitly assuming full pre-merger competition, and consequently assessing how prices change when competition between the firms becomes fully internalized. As an example, let us assume that pre-merger competition between the firms is perfectly internalized through the input price. As prices are optimally chosen pre-merger, a further price increase would not be optimal. Consequently, the merger causes no pricing pressure. However, using the horizontal UPP test, not taking the vertical relation into account, would predict a positive price change. Consequently, an estimation of the UPP, which does not consider vertical relations, will be incorrect.

In this article, we show that the framework easily can be expanded, in order to predict correct pricing pressures for several structural changes. Firstly, we adjust the formula to include potential efficiency gains. Then, we adjust the formulas in order to predict correct pricing pressures if two vertically related firms merge. Finally, we show that the framework also can be used in non-merger situations - by developing a formula for predicting pricing pressures following the introduction of a vertical relation between an integrated firm and a downstream rival.

The rest of this paper is structured as follows. Firstly, we will give a short presentation of the current UPP framework, as presented in Hausman et al. (2011). This will serve as a baseline for the vertical relations model. However, we allow marginal costs to change due to the merger, thus taking into account the possibility for efficiency gains. In section 3, we present the model for the UPP of merging competitors with pre-merger vertical relations. In section 4, the pricing pressure following an exit of the upstream production of one vertically integrated competitor will be analyzed. Section 5 summarizes a comparison of the standard UPP-test to the adjusted formula, using an equilibrium model. Section 6 offers some concluding remarks.

1In the case of two vertically integrated firms, it is reasonable to assume that the two firms will utilize the most efficient production technology post-merger. In this sense, the high cost firm will experience an efficiency gain, as it can produce at a smaller marginal cost post-merger. Such an efficiency gain assumes that the low cost firm has no binding capacity constraints.
2 UPP with efficiency gains

Let us consider a horizontal merger between firm 1 and firm 2. The firms compete in a differentiated market and each produce one product. The firms 1 and 2 have pre-merger profits $\Pi_1$ and $\Pi_2$, respectively:

$$\Pi_1 = (p_1 - c_1^0)Q_1$$
$$\Pi_2 = (p_2 - c_2^0)Q_2$$

where $c_i^0$ represents the pre-merger marginal costs, $Q_i$ the quantity and $p_i$ the price charged by firm $i = 1, 2$. We will assume that the firms maximize their profits, yielding the following first order conditions:

$$Q_1 + (p_1 - c_1^0)\frac{\partial Q_1}{\partial p_1} = 0$$
$$Q_2 + (p_2 - c_2^0)\frac{\partial Q_2}{\partial p_2} = 0$$

Rewriting the first order conditions, we find the following expressions:

$$\frac{\partial Q_1}{\partial p_1} = -\frac{Q_1^0}{p_1^0 - c_1^0} \quad (1)$$
$$\frac{\partial Q_2}{\partial p_2} = -\frac{Q_2^0}{p_2^0 - c_2^0} \quad (2)$$

, where $Q_i^0$ and $p_i^0$ for $i = 1, 2$ represent pre-merger equilibrium quantities and prices. Thus, we have now expressed the non-observable derivatives of demand in terms of observable, pre-merger variables.

After merging, the firms will seek to maximize their joint profit:

$$\Pi_m = \Pi_1 + \Pi_2 = (p_1 - c_1)Q_1 + (p_2 - c_2)Q_2$$
Post-merger first order conditions are thus:

\[ Q_1 + (p_1 - c_1) \frac{\partial Q_1}{\partial p_1} + (p_2 - c_2) \frac{\partial Q_2}{\partial p_1} = 0 \]

\[ (p_1 - c_1) \frac{\partial Q_1}{\partial p_2} + Q_2 + (p_2 - c_2) \frac{\partial Q_2}{\partial p_2} = 0 \]

By rewriting the post-merger first order expressions, we are able to express the pricing pressures in terms of observable parameters. To see this, let us first divide the post-merger first order condition for firm 1 by \( \frac{\partial Q_1}{\partial p_1} \). Rewriting yields:

\[ (p_1 - c_1) \frac{\partial Q_1}{\partial p_1} \]

Assuming that \( \frac{\partial Q_1}{\partial p_1} \) is constant, we can substitute for (1) and find:

\[ (p_1 - c_1) - (p_2 - c_2) D_{12} = - \frac{Q_1}{\partial Q_1 / \partial p_1} (p_1^0 - c_1^0) \]

Let us now assume that prices and marginal costs may change after the merger, such that \( p_i = p_i^0 + \Delta p_i \) and \( c_i = c_i^0 + \Delta c_i \). Substituting for post-merger prices and marginal costs in the post-merger first order condition and dividing through by \( p_i^0 \), we can find that:

\[ \frac{\Delta p_1}{p_1^0} = \frac{p_2^0 - c_2^0 - \Delta c_2}{p_2^0} - (Q_1 - Q_1^0) \frac{(p_1^0 - c_1^0)}{p_1^0 Q_1^0} + \frac{\Delta c_1}{p_1^0} \]

Finally, we can assume that:

\[ Q_1 - Q_1^0 \approx \frac{\partial Q_1}{\partial p_1} \Delta p_1 + \frac{\partial Q_1}{\partial p_2} \Delta p_2 = \frac{\partial Q_1}{\partial p_1} \Delta p_1 - \frac{\partial Q_2}{\partial p_2} \Delta p_2 \]

By substituting for (1) and (2) and rearranging the terms, we have:

\[ 2 \frac{\Delta p_1}{p_1^0} - \left( \frac{p_2^0}{p_1^0} D_{12} + \frac{p_1^0 - c_1^0}{p_2^0} \frac{p_2^0 Q_2^0}{p_1^0 Q_1^0} \right) \frac{\Delta p_2}{p_2^0} = \frac{p_2^0 - c_2^0 - \Delta c_2}{p_2^0} D_{12} + \frac{\Delta c_1}{p_1^0} \]

An analogous solution for the post-merger first order condition of firm 2 yields:

\[ - \left( \frac{p_1^0}{p_2^0} D_{21} + \frac{p_2^0 - c_2^0}{p_1^0 - c_1^0} \frac{p_2^0 Q_1^0}{p_2^0 Q_2^0} D_{21} \right) \frac{\Delta p_1}{p_1^0} + 2 \frac{\Delta p_2}{p_2^0} = \frac{p_1^0 - c_1^0 - \Delta c_1}{p_1^0} D_{21} + \frac{\Delta c_2}{p_2^0} \]

\[ ^2 \text{Using that the diversion ratio is defined as } D_{12} \equiv -\frac{\partial Q_2}{\partial p_1}. \]

\[ ^3 \text{Note that the only difference between (3) and the expression in Hausman et al. (2011) is that we have allowed } \]

\[ \text{costs to change post-merger. Therefore, we get an added term of } \frac{\Delta c_1}{p_1^0} \text{ on the right hand side of the expression.} \]
Using Cramer’s rule, we find the percentage price increase:

\[
\frac{\Delta p_1}{p_1^0} = \frac{2 \left( \frac{p_0^0 - c_0^1}{p_1^0} - \Delta c_2 \right) D_{12} + 2 \Delta c_1 + \frac{p_0^0 - c_0^1}{p_1^0} - \Delta c_1 \right) D_{12} D_{21} + \Delta c_2 D_{12} + \frac{p_0^0 - c_0^1}{p_1^0} D_{12} + \frac{p_0^0 - c_0^1}{p_2^0} \frac{Q_2^0}{Q_1^0} \left( (p_1^0 - c_1^1 - \Delta c_1) D_{21} + \Delta c_2 \right) D_{21}}{4 - 2D_{12}D_{21} - \frac{p_0^0 - c_0^1}{p_2^0} \frac{Q_2^0}{Q_1^0} (D_{21})^2 - \frac{p_0^0 - c_0^1}{p_1^0} \frac{Q_2^0}{Q_1^0} (D_{12})^2}
\]

This percentage price increase indicates the optimal price change for firm 1, assuming that all other competitors of firm 1 and 2 leave their prices unchanged. Hence, we can interpret the expression as firm 1’s one-sided incentive to raise prices. Note that by setting \(\Delta c_1 = \Delta c_2 = 0\), equation (5) becomes equal to the UPP formula presented by Hausman et al. (2011). An analogous solution may be obtained for firm 2.

As an example of the significance of including efficiency gains, assume that the post-merger marginal costs are reduced significantly, such that the post-merger monopoly price is below the pre-merger prices. Thus, the merger will reduce prices. However, if the change in marginal costs is not included in the calculations, the standard UPP analysis will instead wrongly predict a positive, upward pricing pressure.

Let us assess the effect of one firm’s cost-reduction on the other firm’s pricing pressure. Any reduction in firm 2’s marginal cost always reduces pricing pressure for firm 1 if \(\frac{\partial Q_1}{\partial p_2} > \frac{\partial Q_2}{\partial p_1}\). Increases firm 1’s pricing pressure if \(\frac{\partial Q_1}{\partial p_2} < \frac{\partial Q_2}{\partial p_1}\) and does not affect it when \(\frac{\partial Q_1}{\partial p_2} = \frac{\partial Q_2}{\partial p_1}\) (and vice versa for firm 2’s pricing pressure). Intuitively, a reduction in firm 2’s marginal cost has two conflicting effects on firm 1’s price.

Firstly, the reduction in \(c_2\) leads to a higher margin on firm 2’s product. In order to shift sales to the now more profitable product of firm 2, firm 1 has an incentive to lower its price.

Secondly, a reduction in \(c_2\) gives firm 2 an incentive to lower its price (everything else held equal). This, in turn, gives firm 1 an incentive to lower its price as well.

Which of these effects dominates is determined by the cross-price elasticities. If firm 1’s demand is very sensitive to \(p_2\) (i.e. if \(\frac{\partial Q_1}{\partial p_2}\) is large), it implies that a reduction in \(p_2\) has a strong effect on firm 1’s demand. Thus, the incentive to raise \(p_1\) in order to shift even more sales to firm 2 is limited. This effect is augmented if firm 2’s demand is relatively insensitive to \(p_1\). Thus, changing \(p_1\) does not have a large effect on the demand for firm 2’s product. Consequently, firm 1 also lowers its price.

Let us now assess the effect of one firm’s cost-reduction on its own pricing pressure. If \(2 - \frac{\partial Q_2}{\partial p_2} (D_{21})^2 - D_{12} D_{21} \geq 0\), then a reduction in \(c_1\) implies a smaller pricing pressure for firm 1 than the formula of Hausman et al. (2011) predicts. The analogous result can be shown for firm 2. Let us assess whether this condition is likely to hold.

Firstly, let us assume that competition between the firms is fierce. This implies that diversion ratios are large. We also expect large own-price elasticities of demand and the fraction \(\frac{\partial Q_2}{\partial p_2} / \frac{\partial Q_1}{\partial p_1}\) will consequently approach 1. As diversion ratios are smaller than 1, the condition holds.

Secondly, let us assume that competition between the firms is weak. We can consequently expect small own-price elasticities of demand. A small absolute difference in the elasticities can therefore lead to a large relative difference, potentially causing the fraction \(\frac{\partial Q_2}{\partial p_2} / \frac{\partial Q_1}{\partial p_1}\) to become large. However, week competition implies that the diversion ratios approach zero. Consequently, the condition also holds when competition is weak.

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4The rule states that if \(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}\), then \(x = \frac{ed - bf}{ad - bc}\) and \(y = \frac{af - ec}{ad - bc}\).
Thirdly, let us assume that competition is unbalanced, in the sense that $\frac{\partial^2 Q_2}{\partial p_2^2} \gg \frac{\partial^2 Q_1}{\partial p_1^2}$. However, an unbalanced competition implies that $D_{21}$ approaches zero, whereas $D_{12}$ approaches 1.\(^5\) Hence, the condition holds.

We therefore conclude that the condition likely holds, leading to the conclusion that a reduction in a firm’s cost reduces its pricing pressure. Intuitively, a reduction in costs leads to higher margins for the firms. Raising prices thus implies a larger loss of profit from reduced demand, compared to a situation where costs are unchanged. This reduces the firms’ incentives to raise prices.

If two firms produce at different pre-merger marginal costs, it is reasonable to assume that production will shift towards the low-cost firm’s production technology after a merger.\(^6\) Hence, it is a reasonable assumption that the high-cost firm will experience lower marginal costs post-merger. In assessing the potential effects of mergers, it is therefore important to include efficiency gains in the analysis. We have shown that the cost-reduction for one firm under reasonable conditions can cause an underestimation in its competitor’s pricing pressure. Not using the adjusted formulas (taking such efficiency gains into account) can not only be a loss for firms, but may also be to the detriment of consumers.

3 UPP for competitors with vertical relations

Let us now assess a merger between two firms with vertical relations. Pre-merger, firm 1 is vertically integrated, and produces its own input at marginal cost $c_1$. Downstream, it competes with firm 2, to whom it sells the input for a price $w$. This scenario is depicted in figure 2.

In the model, we have implicitly assumed that firm 1 produces inputs at the same marginal cost, regardless if it sells the input or uses it itself. Assuming that $w > c_1$ consequently implies that firm 1 sells the input at a price above marginal cost. As we can assume that a firm’s end-user price is increasing in its costs (and therefore also the input prices), we know that firm 2 subsequently sets a higher price compared to a situation where it buys inputs at a price equal to marginal cost. If firm 1 does in fact sell the input at a price $w > c_1$, competition is partially\(^7\) internalized in the pre-merger situation. This is due to the fact that firm 1 earns a profit on firm 2’s sales, adjusting its own price (and possibly $w$) in order to maximize its pre-merger profit. The formula in Hausman et al. (2011) does not take this fact into account, as the formulas assume full competition between the firms pre-merger.

Let $Q_1, Q_2$ and $p_1, p_2$ denote the quantities sold and prices of firm 1 and firm 2, respectively. Before the merger, the firms maximize the following profits:

$$\Pi_1 = (p_1 - c_1)Q_1 + (w - c_1)Q_2$$
$$\Pi_2 = (p_2 - w)Q_2$$

Thus, firm 1 produces at marginal cost $c_1$, earning a margin of $(p_1 - c_1)$ on its own sales and a

\(^5\)As $D_{21} = \frac{\partial^2 Q_1}{\partial q_2 \partial p_2}$ and $\partial Q_2 / \partial p_2$ is large, $\partial Q_2 / \partial p_2$ will become small.

\(^6\)Assuming that there are no binding upstream capacity constraints.

\(^7\)Note that, given the vertical structures between the firms, it is not possible for them to internalize competition completely. Even if firm 1 chooses $w$ optimally in order to maximize joint profits, the result will be that firm 1 sets a price slightly lower than its post-merger price. Firm 2 will set a pre-merger price which is higher than the post-merger price. Consequently, firm 2 will have a negative pricing pressure if $w$ is set optimally in the pre-merger situation. Despite this being an interesting result, the deliberation of it is beyond the scope (and purpose) of this paper.
margin of \((w - c_1)\) on the sales of firm 2. Firm 2 buys inputs at cost \(w\), thus yielding a margin of \((p_2 - w)\) on its own sales.\(^8\)

First order conditions are:

\[
\frac{\partial Q_1}{\partial p_1} = -\frac{p_1^0 - c_1^0 - (w - c_1^0)D_{12}}{Q_1^0} \quad (6)
\]

\[
\frac{\partial Q_2}{\partial p_2} = -\frac{Q_2^0}{(p_2^0 - w)} \quad (7)
\]

where \(Q_i^0, p_i^0\) for \(i = 1, 2\) again represent pre-merger equilibrium quantities and prices, and \(c_1^0\) represents firm 1’s pre-merger marginal cost. Post-merger profit is the sum of the two firms’ profit functions, and can be expressed as

\[
\Pi = (p_1 - c_1)Q_1 + (p_2 - c_1)Q_2
\]

with first order conditions

\[
(p_1 - c_1) - (p_2 - c_1)D_{12} = -\frac{Q_1}{\partial Q_1}\left(\frac{\partial Q_1}{\partial p_1}\right)
\]

and

\[
(p_2 - c_1) - (p_1 - c_1)D_{21} = -\frac{Q_2}{\partial Q_2}\left(\frac{\partial Q_2}{\partial p_2}\right)
\]

By substituting for the pre-merger first order conditions, we find:

\[
(p_1 - c_1) - (p_2 - c_1)D_{12} = \frac{Q_1\left(p_1^0 - c_1^0 - (w - c_1^0)D_{21}\right)}{Q_1^0} \quad (8)
\]

\(^8\)As a simplification, we assume zero fixed costs. This assumption does not alter the first order conditions, and thus does not affect the optimal allocation of the firms. We also assume that firm two has no marginal costs, other than the wholesale price \(w\). It is possible to adjust the calculations further, in order to include other marginal cost (and other cost-structures). However, the mechanisms presented in this article are still valid.
\[(p_2 - c_1) - (p_1 - c_1)D_{21} = \frac{Q_2(p_2^0 - w)}{Q_2^0}\] (9)

Finally, we can again substitute for \(p_i = p_i^0 + \Delta p_i\) and \(c_1 = c_1^0 + \Delta c_1\), as well as using that

\[\langle Q_i - Q_i^0 \rangle = \frac{\partial Q_i}{\partial p_i} \Delta p_i - \frac{\partial Q_i}{\partial p_j} D_{ji} \Delta p_j\]

to rewrite equations (8) and (9).

Using Cramer’s rule, we find the following pricing pressures:\(^{10}\)

\[\Delta p_1 = \frac{\left(\frac{\Delta c_1}{p_1} + \left(\frac{p_2 - w - \Delta c_1}{p_1} D_{12}\right) 2 + \left(\frac{p_1 D_{12} + Q_2 Q_1}{p_2} D_{21} \left(\frac{p_1 - c_1 - (w - c_1)D_{12}}{p_2 - w}\right) \right) \right) \left(\frac{p_1 - c_1 - \Delta c_1}{p_2} D_{21} - \frac{w - c_1 - \Delta c_1}{p_2}\right)}{4 - \left(\frac{p_1 D_{12} + Q_2 Q_1}{p_2} D_{21} \left(\frac{p_1 - c_1 - (w - c_1)D_{12}}{p_2 - w}\right) \right) \left(\frac{p_2 D_{21} + Q_2 Q_1}{p_2} D_{21} \left(\frac{p_2 - w}{p_2}\right) + \left(\frac{p_1 - c_1 - \Delta c_1}{p_2} D_{21} - \frac{w - c_1 - \Delta c_1}{p_2}\right) \right) \right)}\] (10)

\[\Delta p_2 = \frac{2 \left(\frac{p_1 - c_1 - \Delta c_1}{p_2} D_{21} - \frac{w - c_1 - \Delta c_1}{p_2}\right) + \left(\frac{p_2 D_{21} + Q_2 Q_1}{p_2} D_{21} \left(\frac{p_2 - w}{p_2}\right) + \left(\frac{p_1 - c_1 - \Delta c_1}{p_2} D_{21} - \frac{w - c_1 - \Delta c_1}{p_2}\right) \right) \right) \left(\frac{p_2 D_{21} + Q_2 Q_1}{p_2} D_{21} \left(\frac{p_2 - w}{p_2}\right) + \left(\frac{p_1 - c_1 - \Delta c_1}{p_2} D_{21} - \frac{w - c_1 - \Delta c_1}{p_2}\right) \right) \right)}{4 - \left(\frac{p_1 D_{12} + Q_2 Q_1}{p_2} D_{21} \left(\frac{p_1 - c_1 - (w - c_1)D_{12}}{p_2 - w}\right) \right) \left(\frac{p_1 D_{12} + Q_2 Q_1}{p_2} D_{21} \left(\frac{p_1 - c_1 - (w - c_1)D_{12}}{p_2 - w}\right) \right)}\] (11)

As seen in equations (10) and (11), there no longer is a symmetric solution to the pricing pressure when firms have vertical relations. For a \(w\) sufficiently high, it is even possible that the UPP for firm 2 is negative. This implies that, following the merger, firm 2 will reduce its price, as opposed to raising it like the traditional UPP calculation would suggest.

Intuitively, there are several mechanisms affecting the firms’ pricing pressures. We can assess them in two ways; by assessing the firms’ incentives separately, or by comparing their relative margins. In assessing the firms separately, let us begin by examining firm 2’s costs. Pre-merger, the firm has a marginal cost \(w\). Post-merger, the marginal cost is reduced to \(c_1\). This cost reduction will, as previously discussed, lower firm 2’s incentive to raise its price. As previously discussed, the cost reduction for firm 2 will also affect the pricing pressure of firm 1. Whether this effect is positive or negative, depends on the relationship between the two cross-price elasticities. Furthermore, for firm 1 there is an additional effect. As firm 1 was able to regain some of its profits through the sales of firm 2 , it had incentives to set a higher price in the pre-merger situation. This reduces the pricing pressure for firm 1, compared to the formula by Hausman et al. (2011).

Another mechanism becomes apparent when considering the firms’ pre- and post-merger margin. The UPP test considers the products’ relative profitability as well as the degree of competition between them (measured by diversion rates). A relatively low marginal will yield a low alternative cost for a price increase, as the loss of sales to other products becomes less costly. Conversely, a product with a relatively high margin yields a high alternative cost for a price increase, deeming a further increase more unlikely. If there is a vertical pre-merger relation between the two firms, a standard UPP-test would consider \((p_2 - w)\) the margin of firm 2. However, as \(w\) is larger than the marginal cost of upstream production, the standard UPP-test underestimates the relative size of

\(^9\)The method and substitutions are equivalent to those used in section 2, thus they are not discussed in further detail here.

\(^{10}\)In the expressions, we have made the following simplifications in notation: \(c_i^0 = c_i, p_i^0 = p_i, Q_i^0 = Q_i\) for \(i = 1, 2\).

\(^{11}\)Symmetric in the sense that the expressions for the two firms’ pricing pressure are symmetric. However, if firms are not symmetric, diversion rates, prices and costs will also differ. Consequently, also the firms’ pricing pressure will not be the same.
firm 2’s margin. Consequently, the test also underestimates the alternative cost of raising $p_2$, thus overestimating firm 2’s pricing pressure.

As the standard Hausman UPP-test underestimates the relative margin of firm 2, firm 1 is perceived as having a relatively high profitability. Consequently, the test overestimates the alternative cost of raising $p_1$. This can therefore lead to underestimation of firm 1’s incentive to raise its price. Conversely, as firm 1 was able to regain profit through the sales of firm 2 in the pre-merger situation, the effect of the vertical relation could also lead to an overestimation of firm 1’s pricing pressure. The total effect for firm 1 is therefore not immediately clear. However, the application of the adjusted formulas in (10) and (11) will yield correct pricing pressures.

As an example, consider a situation in which the merging firms have internalized pre-merger competition perfectly through $w$. In that case, there would be no incentive to change prices post-merger. However, the Hausman formula would predict upward pricing pressures for both firms, as it does not take the vertical relation into account. We can conclude that if $w > c_1^0$, then competition between firm 1 and 2 is partially internalized pre-merger. The adjusted formulas presented here can therefore be interpreted as the remaining incentives to raise prices, due to an internalization of the competition, which has not yet been internalized in the pre-merger situation.

4 UPP when one vertically integrated firm terminates upstream production

Let us now take one step back, assessing a scenario where a vertical relation is introduced. Assume that two vertically integrated firms compete in a downstream market. However, one of the downstream firms now chooses to shut down its upstream production, and starts to buy inputs from its competitor. This situation is illustrated in figure 3. The motivation for taking this step back is twofold: Firstly, we are able to show that the method for calculating one-sided pricing pressures developed by Hausman et al. (2011) is applicable even in certain non-merger situations. By correctly representing firms’ profits before and after a structural change in the market, we are able to identify one-sided incentives for price changes. Secondly, this allows us to assess the UPP formulas calculated in the previous section. By considering the introduction of a vertical relation step 1, and a merger with vertical relations (as seen in the previous section) step 2, we can test the result of our model against the traditional Hausman-formula.

![Figure 3: Termination of firm 2’s upstream production](image)

In order to determine pricing pressures in this scenario, we again begin by defining pre-change
profits:
\[\Pi_1 = (p_1 - c_1^0)Q_1\]
\[\Pi_2 = (p_2 - c_2^0)Q_2\]

where \(c_i^0\) is the variable input cost of firm \(i\).

The first-order conditions, which we assume to hold, are then given by:
\[
\begin{align*}
\frac{\partial \Pi_1}{\partial p_1} &= Q_1 + (p_1 - c_1^0) \frac{\partial Q_1}{\partial p_1} = 0 \\
\frac{\partial \Pi_2}{\partial p_2} &= Q_2 + (p_2 - c_2^0) \frac{\partial Q_2}{\partial p_2} = 0
\end{align*}
\]

The solutions to these first-order conditions are the observed market prices, which we denote \(p_0^1\) and \(p_0^2\), respectively. We denote, likewise, the quantity sold to be \(Q_0^1\) and \(Q_0^2\), given the optimal prices. The first-order conditions may now be rewritten into:
\[
\begin{align*}
\frac{\partial Q_1}{\partial p_1} &= -\frac{Q_0^1}{(p_0^1 - c_1^0)} \\
\frac{\partial Q_2}{\partial p_2} &= -\frac{Q_0^2}{(p_0^2 - c_2^0)}
\end{align*}
\]

Note that, around the optimal solutions, we have now obtained expressions for the non-observable derivatives as functions of observable variables. This is equivalent to the first order conditions presented in section 2.

Assume now that firm 2 agrees to buy the input from firm 1, for a unit price equal to \(w\). The profit expressions for the two firms are then given by:
\[\Pi_1 = (p_1 - c_1)Q_1 + (w - c_1)Q_2\]
\[\Pi_2 = (p_2 - w)Q_2\]

Note that we now open for that \(c_1 \neq c_1^0\). The first order-conditions, after the change in structure, are now given by:
\[
\begin{align*}
\frac{\partial \Pi_1}{\partial p_1} &= Q_1 + \frac{\partial Q_1}{\partial p_1}(p_1 - c_1) + \frac{\partial Q_2}{\partial p_1}(w - c_1) = 0 \\
\frac{\partial \Pi_2}{\partial p_2} &= Q_2 + (p_2 - w) \frac{\partial Q_2}{\partial p_2} = 0
\end{align*}
\]
Utilizing the first order conditions from (12) and (13), we can rewrite the expressions in the following way:

\[
(p_1 - c_1) - D_{12}(w - c_1) = \frac{Q_1(p_1^0 - c_1^0)}{Q_1^0}
\]

\[
(p_2 - w) = \frac{Q_2(p_2^0 - c_2^0)}{Q_2^0}
\]

By rewriting these equations as seen in the previous sections\(^{12}\) and applying Cramer’s rule, we find the following pricing pressures:

\[
\Delta p_1 \frac{p_0}{p_1} = 2\left(\frac{\Delta c_1}{p_1} + \frac{w - c_1 - \Delta c_1}{p_1} D_{12}\right) - \frac{Q_2 p_2}{p_1 Q_1} D_{21} \frac{(p_1 - c_1)}{p_2} - \frac{Q_1 p_1}{p_2 Q_2} D_{21} \frac{(p_2 - c_2)}{p_1}
\]

\[
\Delta p_2 \frac{p_0}{p_2} = \frac{2(w - c_2)}{p_2} - \frac{(\Delta c_1 + \frac{w - c_1 - \Delta c_1}{p_1} D_{12})}{4 - D_{21} D_{12}} - \frac{Q_1 p_1}{p_2 Q_2} D_{21} \frac{(p_2 - c_2)}{p_1}
\]

where we again have simplified the expressions by denoting \(p_i^0 = p_i\), \(Q_i^0 = Q_i\) and \(c_i^0 = c_i\) for \(i = 1, 2\). Formulas (14) and (15) consequently yield the theoretical pricing pressures of the structural change in the market. However, it is noteworthy that a practical application requires knowledge of the planned wholesale price \(w\) between the firms. In other words, the formulas require knowledge of \(w\) in the pre-merger situation.

5 Comparing the standard UPP-test to the adjusted formulas

In reality, it is difficult to test the original model of Hausman et al. (2011) against the proposed adjusted model with vertical relations. It is however possible to picture the merger between two vertically integrated firms as a two-step process. In the first step, the vertical relation is introduced through the shut-down of one firm’s upstream production. This is the scenario described in section 4. The second step results in a full merger between the two firms, as in section 3. However, the net result of these two steps should be equivalent to a merger with efficiency gains, as described in section 2.

We have tested each step against an equilibrium model, using a Shubik-Levitan utility function, which is given by:

\[
U = \sum_{i=1}^{2} Q_i - \frac{1}{2} \left(2(1-s) \left(\sum_{i=1}^{2} Q_i\right)^2 + s \sum_{i=1}^{2} Q_i^2\right) - \sum_{i=1}^{2} Q_i p_i
\]

where \(s\) is the degree of differentiation between the products and \(Q_i\) for \(i = 1, 2\) denotes the demand for firm \(i\)’s product. By use of this utility function, we were able to calculate equilibrium prices and quantities in each contingency/market structure,\(^{13}\) and therefore also true pricing pressures for each step.

\(^{12}\)We again substitute for \(c_1 = c_1^0 + \Delta c_1, p_i = p_i^0 + \Delta p_i\) for \(i = 1, 2\) and utilize the expressions for \((Q_i - Q_i^0), i = 1, 2\). Rearranging by the two unknowns \(\Delta p_i / p_i\) and \(\Delta q_i / q_i\) then allows us to use Cramer’s rule.

\(^{13}\)Assuming that consumers maximize utility and firms maximize profits in each contingency/market structure.
As a result, we find that the adjusted pricing pressure formulas in section 2 through 4 (equations (5), (10), (11), (14) and (15)) correctly predict the pricing pressure of the equilibrium model. We were also able to verify that the adjusted formulas are able to predict the correct pricing pressures in a merger with vertical relations, regardless the size of \( w \). Consequently, the estimates are correct whether the wholesale price between the firms was set exogenously or through maximization of joint profits. The implication of this fact is that, as long as \( w \) is observable, the calculations using this formula will be accurate.

6 Conclusion

In this article, we have shown how the method for deriving upward pricing pressure formulas in Hausman et al. (2011) is not limited to horizontal merger, and can be applied to derive adjusted formulas for several structural changes in the market.

Firstly, we derive formulas for the merger between two vertically integrated firms. If we assume that there are no capacity constraints in the upstream production market, it is reasonable to assume that the high-cost firm will produce with the low-cost firm’s production technology in the post-merger situation. Consequently, the high-cost firm will experience a reduction in costs due to the merger. This, in turn leads to a reduction in the high-cost firm’s pricing pressure, compared to a situation where its costs remain unchanged after the merger. Hence, the standard Hausman formula overestimates the pricing pressure of the high cost firm. For the low cost firm, the Hausman formula can either over- or underestimate the pricing pressure, depending on the relationship between the two firms’ cross-price elasticities.

Secondly, we derive adjusted UPP-formulas for a merger between two firms with vertical pre-merger relations. Specifically, we assess a scenario where firm 1 produces upstream inputs, which it sells to its downstream competitor (firm 2). As firm 1 earns a profit on firm 2’s pre-merger sales, competition is already partially internalized in the pre-merger situation. As the standard Hausman-test does not include this fact in the pre-merger profit functions, estimates of the pricing pressures will be incorrect. In order to see this, assume that pre-merger competition is perfectly internalized through the wholesale price. Consequently, there are no pricing pressures due to the merger. The standard Hausman formula would predict positive pricing pressures for both firms, as opposed to the adjusted formulas.

Intuitively, the standard Hausman formula considers the relative margins of the two products. If the wholesale price for the input is larger than the upstream producer’s marginal cost, it leads to an artificially low margin on firm 2’s product. Consequently, the alternative cost of raising \( p_2 \) is underestimated, resulting in a predicted pricing pressure, which is too large. In fact, it is possible that the pricing pressure of firm 2 even becomes negative.

Thirdly, we show that the method of Hausman et al. (2011) also can be utilized in non-merger situations. In example, a vertically integrated firm may terminate its upstream production, consequently buying inputs from a competitor. Hence, a vertical relation is introduced. We show that it is possible to calculate the pricing pressures for the two firms following the structural change in the market. Thus, pricing pressure formulas can also be applied in order to assess the consequences of structural changes in a market. However, in order to predict pricing pressures correctly, the post-change wholesale price between the two firms needs to be observable.

The article provides a set of upward pricing pressure formulas, which are more accurate in predicting pricing pressures in different merger situations and structural changes in the market. As such formulas are used by competition authorities in order to flag potentially harmful mergers,
it is important that the tools in use are as accurate as possible. This article therefore aims at expanding the range of such tools. However, it is important to keep in mind that restrictions of the standard Hausman test also are applicable for the adjusted formulas. Such restrictions include assumptions about demand functions, demand elasticities, diversion ratios and the empirical challenges of determining accurate marginal costs and diversion ratios for the firms involved.

References


