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Abstract

Stepwise models of technological progress described by Philippe Aghion and his co-authors (1997, 2001, 2005) capture the incentives of firms to innovate in order to escape competition and the disincentives from sharing profits with other technological leaders. The models yield intuitively appealing predictions about the effects of competition on innovation, but they are limited to competition in duopolies. This paper extends the models to oligopolies and shows that the predictions of the effects of competition on innovation from the duopoly models do not generalize to oligopolies.

1 Introduction

In a series of papers, Philippe Aghion and his co-authors (Aghion et al. 1997, 2001, 2005) strengthen the theoretical foundation for the analysis of innovation incentives by developing models of "stepwise" innovation. The firms in these models make discrete improvements to their production technologies and a firm that is behind the technological frontier must first catch up to the current market leader before it can become a new leader. Competition has contrasting effects in these models. Competition increases the incentive for a firm to invest to differentiate itself from an equally efficient rival, but deters investment by reducing the profit the firm can earn if it catches up to a market leader. Whether an increase in competition increases or decreases incentives to innovate depends on industry structure and these relative effects.

Aghion and his co-authors apply their theory to a continuum of industries, each of which is populated by a duopoly. They model the effects of competition by allowing the intensity of competition to vary in the duopoly markets. In this paper, we extend the stepwise innovation models in Aghion et al. (2001) and Aghion et al. (2005) to symmetric n-firm oligopolies. We show that the intensity of competition in a duopoly is not a reliable proxy for the effect of greater rivalry.

In particular, for some parameter specifications the innovation rate in the duopoly model of Aghion et al. (2005) is a decreasing function of the intensity of competition. However, with the same specifications the n-firm model shows

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innovation increasing with the number of rivals. For other parameter specifications the duopoly model in Aghion et al. (2001) shows innovation increasing with the intensity of competition, while our model of an n-firm industry shows an "inverted-U" relationship between innovation and the number of rivals.

These results are relevant to antitrust policy for industries that exhibit stepwise innovation. Antitrust authorities often take innovation into account in their enforcement decisions for mergers in high technology industries (Hesse, 2014; Gilbert and Greene, 2015). Our analysis demonstrates that existing models of stepwise innovation have to be generalized to oligopolies for the theory to inform merger policy for industries in which innovation progresses in a stepwise fashion.[2]

Section 2 develops the value functions for firms in a symmetric $n-$firm oligopoly and derives investment rates under the assumption that firms' production technologies can differ by at most one increment of marginal cost. The value functions and investment rates depend on whether firms are leaders at the technological frontier or laggards with an inferior technology. Section 3 develops general formulas for the expected rate of technological innovation averaged over all industries with different numbers of leaders and laggards. In section 4 we estimate oligopoly prices and profits corresponding to the demand specifications in Aghion et al. (2001) and Aghion et al. (2005). Section 5 applies these estimates to compare oligopoly innovation rates for oligopolies to the estimated innovation rates in the duopoly models with corresponding demand parameters. Section 6 concludes.

2 Stepwise investment with n rivals

Following Aghion et al. (2001, 2005) we model incentives for firms to invest to lower their marginal production costs. Investment generates a Poisson hazard rate of discovery at a cost that is proportional to the square of the hazard rate. Successful investment reduces a firm's marginal cost by one "step" from c to c/γ with $\gamma > 1$. The model makes the simplifying assumption that the industry elasticity of demand is unity, which implies that profits and therefore the incentives to invest depend only on relative costs.

We develop the model under the assumption that rival firms' technologies can differ by at most one step.[3] A firm can be a "leader" at the technological frontier or a "laggard" that is one step behind. (Firms are "neck-and-neck" if all firms are leaders.) We begin by deriving the equations that determine the values of leaders and laggards in an $n-$firm oligopoly conditional on the number of leaders.

[2] Measures that affect competition holding the number of rivals fixed can be relevant for antitrust policy in a non-merger context. See, e.g., Segal and Whinston (2007) and Baker (2016).

[3] The dependence of the innovation rate on the number of rival firms is not qualitatively different if we allow firms' technologies to differ by at most two steps.

Define:

- k = the number of leaders
- π_k = a leader's profit flow with k leaders and $n-k$ laggards
- $\bar{\pi}_k$ = a laggard's profit flow
- V_k = a leader's present-value profit
- $\bar{V}_k$ = a laggard's present-value profit
- x_k = a leader's investment rate
- $\bar{x}_k$ = a laggard's investment rate

Firms are motivated to invest in research and development by the change in profits if their investments are successful and by capital gains or losses that would occur if they or their rivals innovate. The value function for a firm that is one of k leaders with $n-k$ firms that lag by one step is

$$V_k = \begin{array}{c} (\pi_k - \frac{1}{2}\beta x_k^2)dt + e^{-rdt}\{x_k dt V_1 + (k-1)x'_k dt \bar{V}_1 + (n-k)\bar{x}_k dt V_{k+1} \\ +[1-(x_k + (k-1)x'_k + (n-k)\bar{x}_k)dt]V_k\} \end{array}$$

where x'_k is investment by a rival leader. The first term is the firm's current profit net of its investment cost. The other terms are present-value profits weighted by the probability of transitioning to different industry states. The stepwise model assumes that firms can advance the state of technology by only one step if they invest in research and development and firms cannot lag the market leader by more than one step. As a consequence, if a market leader advances by one step, all other firms lag the leader by one step, regardless of whether they were at the industry frontier or one step behind when the firm advanced.

Spillovers are built into the stepwise model in the sense that firms cannot lag the market leader by more than one innovation step. In addition, we allow lagging firms to benefit from additional technological spillovers that reduce the marginal cost of investing from $\beta\bar{x}_k$ to $s\beta\bar{x}_k$ with $s \in (0,1]$.

The value function for a laggard when k rivals are at the technology frontier is

$$\bar{V}_k = \begin{array}{c} (\bar{\pi}_k - \frac{1}{2}s\beta\bar{x}_k^2)dt + e^{-rdt}\{\bar{x}_k dt V_{k+1} + (n-k-1)x'_k dt \bar{V}_{k+1} + k x_k dt \bar{V}_1 \\ +[1-(\bar{x}_k + (n-k-1)\bar{x}'_k + k x_k)dt]\bar{V}_k\} \end{array}$$

where $\bar{x}'_k$ refers to a rival laggard's investment. Note that V_k is defined for $k = 1, ..., n$ and $\bar{V}_k$ for $k = 1, ..., n-1$. Moreover, note that $x_1 = 0$ because a firm that is the only market leader cannot further distance itself from its rivals.

Taking limits with $e^{-rt} \simeq 1 - rdt$ and ignoring second-order terms gives the Bellman equations

$$\begin{aligned} rV_k &= \pi_k + x_k (V_1 - V_k) + (k-1) x'_k \left(\bar{V}_1 - V_k\right) \\ &\quad + (n-k)\bar{x}_k (V_{k+1} - V_k) - \frac{1}{2}\beta x_k^2 \end{aligned} \tag{1}$$

and

$$\begin{aligned} r\bar{V}_k &= \bar{\pi}_k + \bar{x}_k \left(V_{k+1} - \bar{V}_k\right) + (n-k-1) \bar{x}'_k \left(\bar{V}_{k+1} - \bar{V}_k\right) \\ &\quad + kx_k \left(\bar{V}_1 - \bar{V}_k\right) - \frac{1}{2} s\beta \bar{x}_k^2. \end{aligned} \tag{2}$$

The term $x_k (V_1 - V_k)$ in equation (1) is the expected capital gain from successful investment by a leader when the industry has a total of k leaders. This term measures the value to a leader from escaping competition and is the inverse of the Arrow replacement effect (Arrow, 1962). The greater the competition, the greater is the incentive to invest (a smaller Arrow replacement effect).

The term $\bar{x}_k \left(V_{k+1} - \bar{V}_k\right)$ in equation (2) is the expected capital gain from successful investment by a laggard in an industry with k leaders. With typical profit specifications, the larger the number of existing leaders, the more difficult it is for a new leader to achieve an immediate return from its innovation because $\pi_{k+1} - \bar{\pi}_k$ is declining in k. This term measures the magnitude of Schumpeterian appropriation effects (Schumpeter, 1942). However, competition also increases the firm's incentive to invest if it catches up to market leaders. As $V_{k+1} - \bar{V}_k$ incorporates both effects, the net effect of competition from industry leaders for laggard investment is uncertain.

Other terms in equations (1) and (2) reflect capital losses when rivals innovate. For example, the term $(k-1) x'_k \left(\bar{V}_1 - V_k\right)$ is a leader's expected capital loss from innovation by rival leaders and the term $(n-k)\bar{x}_k (V_{k+1} - V_k)$ is a leader's expected capital loss from innovation by laggards.

The profit-maximizing investment rates satisfy the first-order conditions

$$x_k^* = \frac{1}{\beta}(V_1 - V_k) \tag{3}$$

and

$$\bar{x}_k^* = \frac{1}{s\beta} \left(V_{k+1} - \bar{V}_k\right). \tag{4}$$

Equations (1)-(4) enable closed-form solutions for the profit-maximizing investment rates. The steps are in Appendix 1. Leader investments satisfy

$$\begin{aligned} rx_k &= \frac{1}{\beta}(\pi_1 - \pi_k) - (n-1)\bar{x}_1 x_2 \\ &\quad - (k-1) x_k \left[-s\bar{x}_1 - x_2 + x_k\right] \\ &\quad - (n-k) \bar{x}_k \left[-x_{k+1} + x_k\right] - \frac{1}{2} x_k^2 \end{aligned} \tag{5}$$

for $x_k = 1, ..., n$ and laggard investments satisfy

$$\begin{aligned} sr\bar{x}_k &= \frac{1}{\beta}(\pi_{k+1} - \bar{\pi}_k) + kx_{k+1}\left[-s\bar{x}_1 - x_2 + x_{k+1}\right] \\ &+ (n-k-1)\,\bar{x}_{k+1}\left[-x_{k+2} + x_{k+1}\right] + \frac{1}{2}x_{k+1}^2 \\ &- (n-k-1)\,\bar{x}_k\left[-s\bar{x}_{k+1} - x_{k+2} + x_{k+1} + s\bar{x}_k\right] \\ &- kx_k\left[-s\bar{x}_1 - x_2 + x_{k+1} + s\bar{x}_k\right] - \frac{1}{2}s\bar{x}_k^2 \end{aligned} \tag{6}$$

for $\bar{x}_k = 1, ..., n-1$. We use numerical methods to solve these equations for the profit-maximizing investment rates.

The corresponding investment rates in Aghion et al. (2001) are

$$(r+h)x_2 = \frac{1}{\beta}(\pi_1 - \pi_2) - \frac{1}{2}x_2^2$$

and

$$(r+h)\bar{x}_1 = \frac{1}{\beta}(\pi_2 - \bar{\pi}_1) - x_2\bar{x}_1 + \frac{1}{2}x_2^2 - \frac{1}{2}\bar{x}_1^2.$$

These equations are identical to the corresponding equations (5) and (6) when $n = 2$, with the exception that we model technological spillovers with the cost parameter $s \in (0, 1]$ and assume that $h = 0$.[4] We depart from the spillover assumptions in Aghion et al. (2001, 2005) because, unlike our formulation, their additive specification implies that the proportional effect of spillovers on each firm's investment is an increasing function of the number of rivals. That has no consequence when the number of rivals is fixed, but assigns an unreasonable weight to spillovers for industries with larger numbers of rivals.[5]

3 Innovation rates

Aghion et al. (2001, 2005) model innovation by a continuum of industries, each of which is a duopoly. The average innovation rate over all industries depends on the share of industries in which one firm is at the technological frontier, with the remainder neck-and-neck, and the investment rates by firms in these states. We extend these models to a continuum of industries, each of which is an oligopoly with n rivals.[6]

[4] These equations also correspond to the closed-form solutions in Aghion et al. (2005) when $h = 0$.

[5] Nonetheless, we obtain qualitatively similar results if we replace the cost parameter, s, with the additive spillover, h, and in an alternative formulation in which the additive spillover rate is independent of the numer of rivals.

[6] For public policy such as merger enforcement, it is more realistic to consider a single industry that can be in different technology states. The asymptotic frequencies of states with a given number of technology leaders correspond to the shares of industries with the same number of leaders in a continuum of industries.

Let k denote a state with k frontier firms and let μ_k be the share of industries in state k. In a steady-state equilibrium, the share of industries that enter state k is equal to the share of industries that exit state k. For state $k = n$,

$$n\mu_n x_n = \mu_{n-1}\bar{x}_{n-1}. \tag{7}$$

The left-hand side of equation (7) is the rate at which industries exit state $k = n$. A share μ_n of industries have $k = n$ and each of the n firms in this state invests at a rate x_n. The right-hand side is the flow into state $k = n$. That flow corresponds to a share μ_{n-1} of industries in which there is a single laggard that invests $\bar{x}_{n-1}$. Similarly, for all other industry states

$$\begin{aligned}
\mu_{n-1}\left[(n-1)\,x_{n-1}+\bar{x}_{n-1}\right] &= 2\mu_{n-2}\bar{x}_{n-2} \\
\mu_{n-2}\left[(n-2)\,x_{n-2}+2\bar{x}_{n-2}\right] &= 3\mu_{n-3}\bar{x}_{n-3} \\
&\;\;\;... \\
\mu_3\left[3x_3+(n-3)\,\bar{x}_3\right] &= (n-2)\,\mu_2\bar{x}_2 \\
\mu_2\left[2x_2+(n-2)\,\bar{x}_2\right] &= (n-1)\,\mu_1\bar{x}_1
\end{aligned}$$

In addition,

$$\sum_{i=1}^{n}\mu_i = 1.$$

Cost falls by the factor $\gamma > 1$ when all firms in the industry advance by one step; i.e., the industry completes a cycle, returning firms to the same state but advanced one step. To derive the average innovation rate across all industries, consider an industry in state 1 with $k = 1$ and label all states with $k > 1$ as state 0. The flow out of state 1 (and into state 0) is $(n-1)\,\mu_1\bar{x}_1$. This is the intensity of laggard investments in state 1 industries. The flow out of state 0 (and into state 1) is (recall that $x_1 = 0$)

$$2\mu_2 x_2 + 3\mu_3 x_3 + .. + n\mu_n x_n = \sum_{i=2}^{n} i\mu_i x_i.$$

An industry advances by one step when it goes through a cycle represented by states 1, 0, and 1. The cycle ends with each firm in the industry one step higher up on the innovation ladder, reducing cost by the factor γ. Hence it is the frequency of cycles such as 1-0-1 that determines the steady state growth rate.

In steady state, at each instant of time there are laggard innovations in state 1 industries with intensity $(n-1)\,\mu_1\bar{x}_1$. Similarly, with the same intensity $\sum_{i=2}^{n} i\mu_i x_i$, there are leader innovations in state 0 industries. As the μ vector is constant in steady state, these flows advance the technology by γ at a rate equal to this intensity.[7] This means that the steady state growth rate is

$$g = (n-1)\,\mu_1 \bar{x}_1 \ln\gamma = \sum_{i=2}^{n} i\mu_i x_i \ln\gamma.$$

4 Demand and profits

We simulate profit-maximizing investments and steady state innovation rates under two demand specifications. The first assumes a constant and equal elasticity of substitution between products offered by the firms in the industry. This specification extends the results in Aghion et al. (2001) to an industry with n firms. The second assumes homogeneous products and extends the modified Bertrand profit function in Aghion et al. (2005).

4.1 Constant elasticity of substitution demand

Consumers maximize

$$u(q_1,...,q_n) = \left(\sum_{j=1}^{n} q_j^{\alpha}\right)^{1/\alpha}$$

subject to

$$\sum_{j=1}^{n} p_j q_j = 1.$$

The parameter $\alpha \in (0,1]$ measures the degree of substitution between products and is an index for the intensity of competition in an industry conditional on the number of rival firms. Utility maximization implies demands

$$q_i = \left(\sum_{j=1}^{n}\left(\frac{p_i}{p_j}\right)^{\frac{\alpha}{1-\alpha}}\right)^{-1}\frac{1}{p_i}.$$

Let p_k be the price charged by a leader when the industry has k leaders and let $\bar{p}_k$ be the corresponding laggard price. Let i denote a leader and j denote a laggard. Define the relative price

$$\hat{p}_k = \frac{p_k}{\bar{p}_k} \quad \text{for } k = 1,...,n-1$$

with $\hat{p}_n = 1$. We show in the Appendix that $\hat{p}_k$ is the solution to

$$\hat{p}_k = \frac{1+\frac{(1-\alpha)}{k+(n-k)(\hat{p}_k)^{\frac{\alpha}{1-\alpha}}-1}}{1+\frac{(1-\alpha)}{k\left(\frac{1}{\hat{p}_k}\right)^{\frac{\alpha}{1-\alpha}}+n-k-1}}\left(\frac{1}{\gamma}\right), \qquad k = 1,2,..,n-1$$

[7] There are also internal flows in state 0 industries. If a laggard innovates in an industry with $k=2$, then k increases to 3, and so on. However, in steady state, these internal flows do not change the composition of firms in state 0 and hence have no impact on growth.

and

$$\pi_k = \frac{1-\alpha}{k+(n-k)\left(\hat{p}_k\right)^{\frac{\alpha}{1-\alpha}}-\alpha}, \qquad k=1,2,..,n$$
$$\bar{\pi}_k = \frac{1-\alpha}{k\left(\frac{1}{\hat{p}_k}\right)^{\frac{\alpha}{1-\alpha}}+n-k-\alpha}. \qquad k=1,2,..,n-1.$$

Note that this specification is identical to the CES demand model in Aghion et al. (2001) when $n=2$.

4.2 Modified Bertrand competition

An alternative specification extends the model of Bertrand competition with homogeneous demand in Aghion et al. (2005). Leading firms share profits that depend on the degree of competition and lagging firms earn zero profits.

For $k=1$,

$$\pi_1 = \pi = 1 - \frac{1}{\gamma}.$$

For $k=2,...,n$,

$$\pi_k = \frac{\pi_1}{k}(1-\alpha)$$

and for $k=1,...,n-1$,

$$\bar{\pi}_k = 0.$$

Only firms at the frontier can earn profits in this specification, which they share equally with other leaders. Profits are inversely related to $\alpha \in [0,1)$ when $k \geq 2$. The term α serves as a measure of the intensity of competition, with the additional assumptions that lagging firms earn zero profits and the profit of a single leader is independent of the number of laggards and the parameter α. This specification is identical to the model of modified Bertrand competition in Aghion et al. (2005) when $n=2$.

5 Results

In this section we present some results of our n-firm oligopoly model for the different demand specifications. We contrast our results with those in Aghion et al. (2001) and Aghion et al. (2005), which describe stepwise innovation in a duopoly with constant elasticity of substitution (CES) demand and modified Bertrand competition, respectively. Although our model is similar in many respects, the results differ substantially, as do their implications for policies such as antitrust enforcement for mergers that may affect innovation incentives. Below we summarize some of the key differences.

In the Aghion et al. (2001) analysis of a duopoly with CES demand, the authors find that the industry innovation rate is typically an increasing function of the degree of competition as measured by α, which is proportional to the elasticity of substitution between products.[8] We also find that the innovation rate is increasing in α if the number of firms is fixed. However, holding α fixed, the innovation rate increases with the number of rivals when n is small and then levels off or decreases. Competition, as measured by the number of rivals holding the elasticity of substitution fixed, has rapidly diminishing returns for innovation and the industry innovation rate has an "inverted-U" dependence on the number of rivals.

We illustrate this result in Figures 1 and 2. The calculations are based on parameters chosen to generate innovation rates consistent with typical industry averages. The results in Figures 1 and 2 are for CES demand with no technological spillovers ($s = 1$). Our focus is on a comparison of the effects of competition on innovation rates when $n = 2$ with the effects for larger values of n. The line for $n = 2$ in Figure 1 corresponds to the CES demand model in Aghion et al. (2001). It is increasing in α; a change in α from 0.10 to 0.90 increases the innovation rate by a factor of about four. In contrast, increasing the number of rivals has a relatively small and nonlinear effect on the innovation rate for any value of α and this small effect changes sign for large values of α.

Figure 2 shows the percentage reduction in the rate of innovation if the number of firms decreases by one. The two series correspond to moderate ($\alpha = .50$) and intense ($\alpha = .95$) market competition as measured by the elasticity of substitution between products. For example, a reduction in the number of rivals from three to two reduces the rate of innovation by about 16% when $\alpha = .50$ and by about 20% when $\alpha = .95$. A reduction in the number of rivals has smaller effects in industries with larger numbers of firms. With CES demand, the innovation rate peaks at three firms if $\alpha = .50$ and at five firms if $\alpha = .95$. This differs sharply from the effect of competition on innovation in the duopoly model, for which competition is measured by the elasticity of substitution rather than the number of rivals.

Figure 3 repeats the comparison under the assumption that spillovers are large ($s = .1$), corresponding to a 90% reduction in the cost of innovation for laggard firms. For a duopoly, spillovers tend to moderate the effect of competition as measured by α, compared to a duopoly with no spillovers. In contrast, holding α fixed, reducing the number of rivals has a larger negative effect on the innovation rate when spillovers are large for an industry with few firms, compared to an industry with no spillovers. Similar to the case with no spillovers, reducing the number of rivals from n to $n - 1$ has a relatively modest effect on the rate of innovation when n is five or more.

With homogeneous products and modified Bertrand competition, Aghion et al. (2005) make the case for an "inverted-U" dependence of innovation on competition. That result depends critically on assumed technological spillovers for a firm that is behind the technological frontier. With Bertrand competition

[8] See Aghion et al. (2001), Figure 6, p. 487.

and no spillovers, the innovation rate in a duopoly is a decreasing function of the degree of competition if the interest rate is not too large.[9] This is evident from Figure 4, which shows the innovation rate as a function of the competition parameter α for different numbers of competitors. The Aghion et al. (2005) model corresponds to $n = 2$. Increasing α from 0.10 to 0.90 lowers the duopoly innovation rate by about 10% when there are no spillovers.

However, Figure 4 also shows that competition increases the rate of innovation in an industry with homogenous demand and modified Bertrand pricing when competition is measured by the number of rivals, even if there are no technological spillovers. Figure 5 shows the percentage reduction in the industry innovation rate when the number of firms in the industry is reduced from n to $n-1$ and $s = 1$, corresponding to no spillovers. In the modified Bertrand model, a reduction in the number of rivals, for example by merger, can have a significant adverse effect on industry innovation even in an industry with as many as five or six firms prior to the reduction in rivalry.

Figure 6 illustrates the effects of technological spillovers in the stepwise innovation model when firms behave as modified Bertrand competitors with homogeneous demand and the spillover cost parameter is $s = .4$. For the duopoly case considered by Aghion et al. (2005) the industry innovation rate exhibits an inverted-U with respect to the competition parameter, α. Increasing α from 0.20 to 0.50 increases the duopoly innovation rate by about 5%, and the rate levels off and then declines for higher values of α.[10] In contrast, adding another rival has a large positive effect on the industry innovation rate when the industry has two or three firms. As in Figure 5, the incremental effect of rivalry on innovation is a decreasing function of the number of firms in the industry.

The reader may question why the qualitative effects of the number of rivals on industry innovation should be so different under the two demand specifications. The reason is that investment rates depend on the incremental profits that a firm can earn by investing in R&D. Innovation by an industry leader increases the firm's instantaneous profit flow by $\pi_1 - \pi_k$ and for a firm that is behind the technological frontier the increase is $\pi_{k+1} - \bar{\pi}_k$. These incremental profits differ significantly for the two demand specifications.[11]

6 Conclusions

The stepwise innovation model developed in a number of papers by Philippe Aghion and his co-authors is instructive because it explicitly accounts for com-

[9] This follows from Proposition 2 in Aghion et al. (2005) with the spillover parameter $h = 0$.

[10] For a duopoly, the inverted-U dependence of the innovation rate on the competition parameter, α occurs only for intermediate values of the spillover cost parameter, s. The duopoly innovation rate decreases with α when spillovers are small (s close to one), as shown in Figure 4, and increases with α when spillovers are large (s close to zero).

[11] See Boone (2000) for a discussion of the competitive implications of different profit functions for R&D investment in a static context.

petitive effects that depend on the relative technological position of a firm in an industry. Competition encourages firms to invest to escape competition from equally efficient rivals, but competition also discourages investment by reducing the payoff from catching up with rivals. However, these papers consider only duopolies and model competitive effects by varying the degree of competition in the duopoly context. We show that the predictions from these models do not generalize to situations in which competition is determined by the number of rivals. In particular, the duopoly assumption is too restrictive to aid public policy for merger enforcement for innovative industries, which addresses the effect of a decrease in the number of rivals in an oligopolistic industry on firms' incentives to innovate and has only indirect effects, if any, on the elasticity of substitution between products.

Our empirical simulations demonstrate that the effects of the number of rivals on incentives to invest in research and development depend critically on the specification of industry demand. Innovation incentives depend on the incremental profits from innovating, which depend on the nature of industry demand and the structure of competition. With homogeneous products and modified Bertrand competition we show that, holding the intensity of competition fixed, increasing the number of rivals in an industry typically increases the innovation rate. In contrast, in the duopoly model described in Aghion et al. (2005) with a similar demand and profit structure, increasing the intensity of competition reduces the innovation rate if there are no technological spillovers. In an industry characterized by constant elasticity of demand, the innovation rate increases with competition in the duopoly model in Aghion et al. (2001). Yet we find that innovation exhibits an inverted-U dependence on the number of firms, holding the elasticity of substitution fixed.

Although we extend the step-wise model to allow for oligopolistic interactions, we do not imply that the model is a general description of innovation incentives. The stepwise model makes numerous assumptions that need not be satisfied in actual markets. Furthermore, our simulations do not account for efficiencies that can be present in many industry circumstances. Industry consolidation can eliminate redundant research and development activities and facilitate benefits from complementary R&D programs. In addition, the merging parties could have complementary products or assets such as distribution facilities that increase the return to R&D activities. These factors may be relevant to an assessment of the effects of a proposed merger on innovation and are not captured in the stepwise innovation models examined in this paper.

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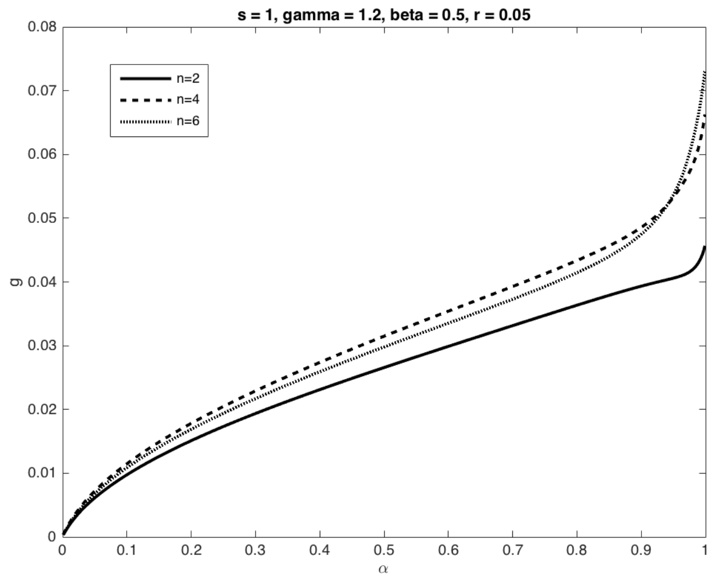

Figure 1. Innovation rates with CES demand (no spillovers: $s = 1$).

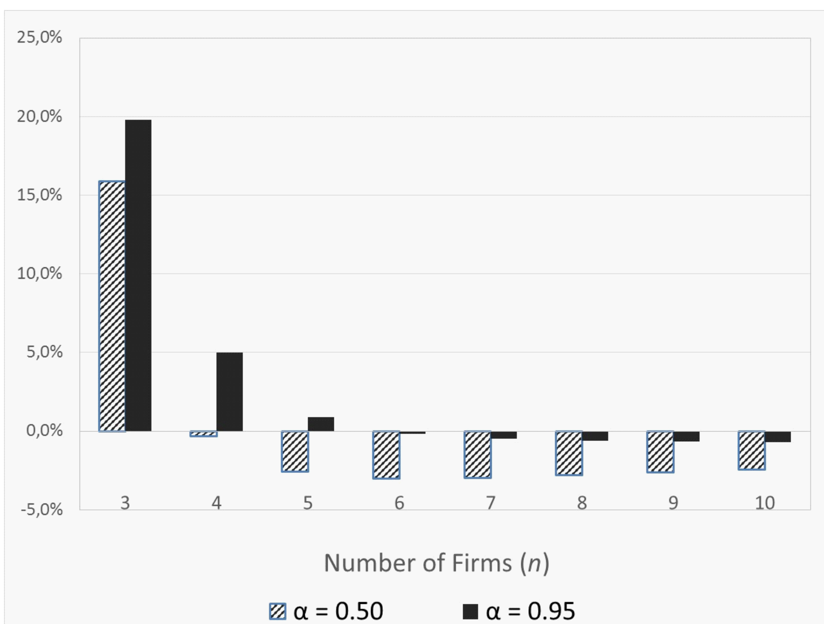

Figure 2. Percent reduction in rate of innovation moving from n to $n-1$ firms with CES demand for different values of the competition parameter (no spillovers: $s = 1$).

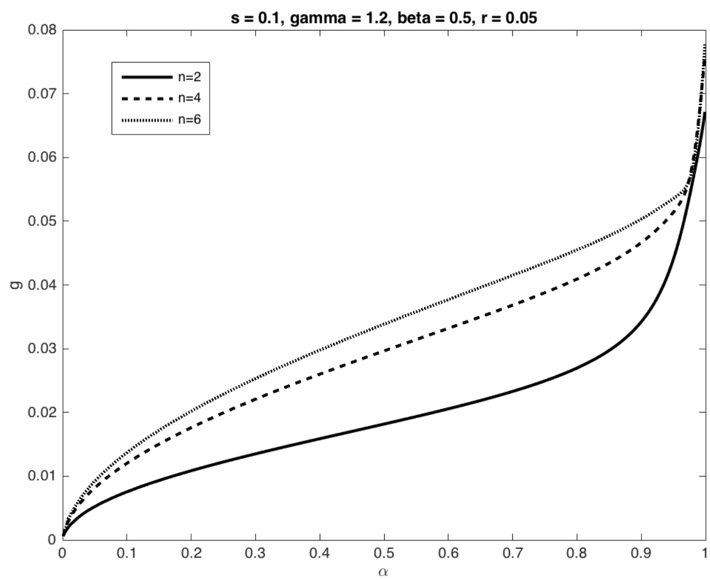

Figure 3. Innovation rates with CES demand and large spillovers ($s = .1$).

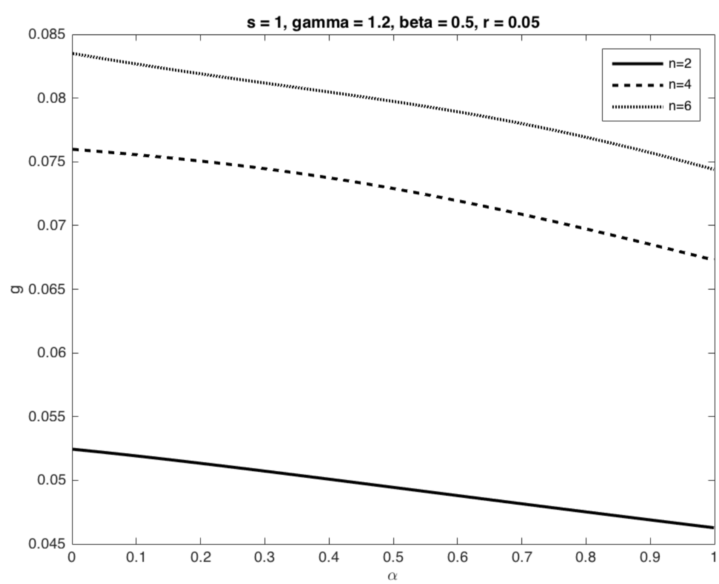

Figure 4. Innovation rates with homogeneous products and modified Bertrand competition (no spillovers: $s = 1$).

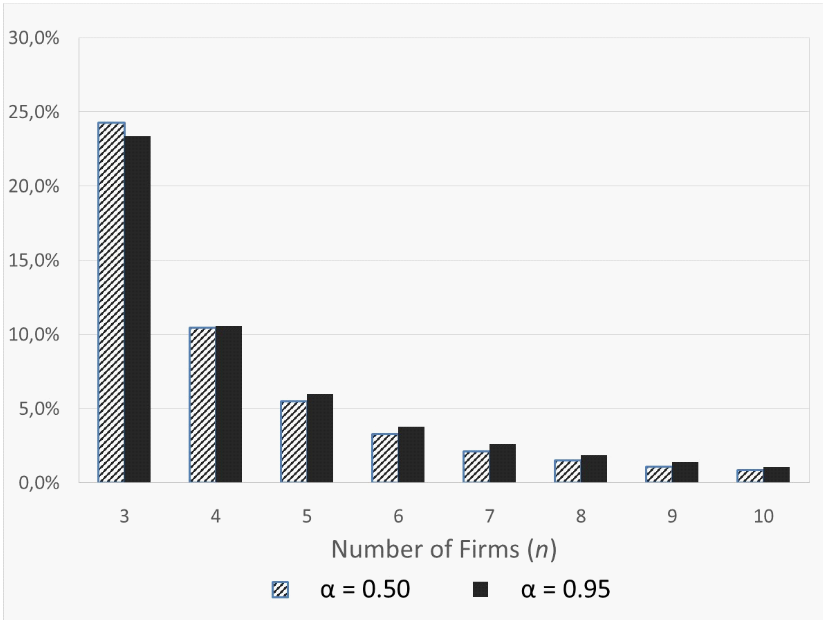

Figure 5. Percent reduction in rate of innovation moving from n to $n-1$ firms with modified Bertrand competition for different values of the competition parameter (no spillovers: $s = 1$).

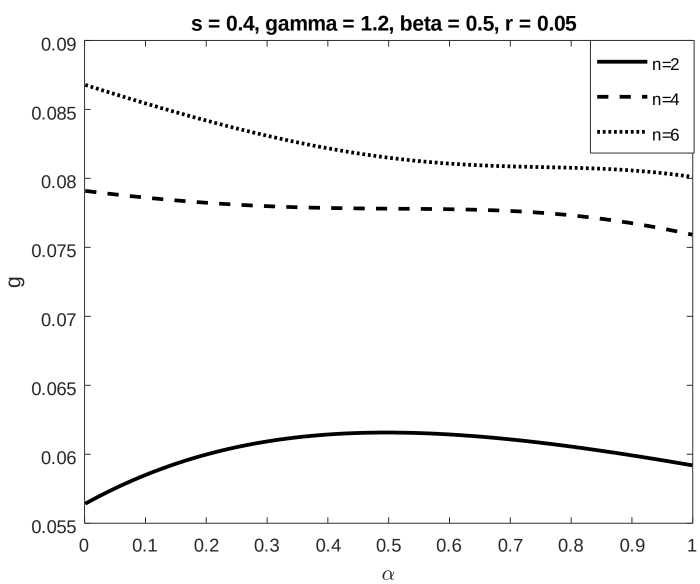

Figure 6. Innovation rates with homogeneous products and modified Bertrand competition (large spillovers: $s = .4$).

Appendix 1. Derivation of investment rates

Closed-form solutions for the profit-maximizing investment rates can be obtained as follows. For x_k, take the difference $r(V_1 - V_k)$. Using $x_1^* = 0$ and the first-order condition $x_k^* = \frac{V_1 - V_k}{\beta}$ gives (omitting the asterisks):

$$\begin{aligned} r\left(V_1 - V_k\right) &= \beta r x_k \\ &= \pi_1 - \pi_k + (n-1)\,\bar{x}_1\left(V_2 - V_1\right) \\ &\quad -x_k\left(V_1 - V_k\right) - (k-1)\,x_k'\left(\bar{V}_1 - V_k\right) \\ &\quad -(n-k)\,\bar{x}_k\left(V_{k+1} - V_k\right) + \frac{1}{2}\beta x_k^2. \end{aligned}$$

Next, we apply the first-order conditions to eliminate the value differences. For example

$$V_2 - V_1 = -\beta x_2$$

and

$$\begin{aligned} \bar{V}_1 - V_k &= \bar{V}_1 - V_2 + V_2 - V_1 + V_1 - V_k \\ &= \beta\left(-s\bar{x}_1 - x_2 + x_k\right). \end{aligned}$$

The result is

$$\begin{aligned} r x_k &= \frac{1}{\beta}(\pi_1 - \pi_k) - (n-1)\,\bar{x}_1 x_2 - x_k^2 \\ &\quad -(k-1)\,x_k\left[-s\bar{x}_1 - x_2 + x_k\right] \\ &\quad -(n-k)\,\bar{x}_k\left[-x_{k+1} + x_k\right] + \frac{1}{2}x_k^2 \end{aligned}$$

for $x_k = 1, ..., n$.

Similarly, for $\bar{x}_k$, take the difference $r\left(V_{k+1} - \bar{V}_k\right)$ and use the first order condition $\bar{x}_k = \frac{V_{k+1} - \bar{V}_k}{s\beta}$:

$$\begin{aligned} r\left(V_{k+1} - \bar{V}_k\right) &= s\beta r\bar{x}_k \\ &= \pi_{k+1} - \bar{\pi}_k + k x_{k+1}\left(\bar{V}_1 - V_{k+1}\right) \\ &\quad +(n-k-1)\,\bar{x}_{k+1}\left(V_{k+2} - V_{k+1}\right) + \frac{1}{2}\beta x_{k+1}^2 \\ &\quad -(n-k-1)\,\bar{x}_k\left(\bar{V}_{k+1} - \bar{V}_k\right) \\ &\quad -k x_k\left(\bar{V}_1 - \bar{V}_k\right) - \frac{1}{2}s\beta\bar{x}_k^2. \end{aligned}$$

Applying other first-order conditions to eliminate the value differences yields

$$\begin{aligned} sr\bar{x}_k &= \frac{1}{\beta}(\pi_{k+1} - \bar{\pi}_k) + k x_{k+1}\left[-s\bar{x}_1 - x_2 + x_{k+1}\right] \\ &\quad +(n-k-1)\,\bar{x}_{k+1}\left[-x_{k+2} + x_{k+1}\right] + \frac{1}{2}x_{k+1}^2 \\ &\quad -(n-k-1)\,\bar{x}_k\left[-s\bar{x}_{k+1} - x_{k+2} + x_{k+1} + s\bar{x}_k\right] \\ &\quad -k x_k\left[-s\bar{x}_1 - x_2 + x_{k+1} + s\bar{x}_k\right] - \frac{1}{2}s\bar{x}_k^2 \end{aligned}$$

for $\bar{x}_k = 1, ..., n-1$.

Appendix 2. Derivation of prices and profits for CES demand

Households maximize

$$f(q_1, ..., q_n) = (q_1^\alpha + .. + q_n^\alpha)^{1/\alpha}, \quad \alpha \in (0, 1]$$

subject to the budget constraint

$$\sum_{i=1}^{n} p_i q_i = 1$$

The associated first order conditions are (μ is the lagrange parameter)

$$(q_1^\alpha + .. + q_n^\alpha)^{1/\alpha} \, q_j^{\alpha-1} - \mu p_j = 0$$

thus

$$\left(\frac{q_j}{q_i}\right)^{\alpha-1} = \frac{p_j}{p_i}.$$

Using this result along with the budget constraint gives

$$q_j = \left(\sum_{i=1}^{n} \left(\frac{p_j}{p_i}\right)^{\frac{\alpha}{1-\alpha}}\right)^{-1} \frac{1}{p_j}.$$

Following Aghion et al. (2001), let

$$\lambda_j = p_j q_j = \left(\sum_{i=1}^{n} \left(\frac{p_j}{p_i}\right)^{\frac{\alpha}{1-\alpha}}\right)^{-1}.$$

Hence the demand elasticity for product j is:

$$\begin{aligned}
-\eta_j &= \frac{\alpha}{1-\alpha} \frac{\sum_{i=1}^{n} \left(\frac{p_j}{p_i}\right)^{\frac{\alpha}{1-\alpha}} - 1}{\sum_{i=1}^{n} \left(\frac{p_j}{p_i}\right)^{\frac{\alpha}{1-\alpha}}} + 1 \\
&= \frac{\alpha}{1-\alpha} \left[\frac{1}{\lambda_j} - 1\right] \lambda_j + 1 \\
&= \frac{1 - \lambda_j \alpha}{1 - \alpha}.
\end{aligned}$$

The first order condition for profit maximization is

$$p_j \left(1 + \frac{1}{\eta_j}\right) = c_j$$

or

$$p_j \alpha \left(\frac{1-\lambda_j}{1-\lambda_j \alpha} \right) = c_j$$

so that

$$p_j = \left(\frac{1-\alpha\lambda_j}{\alpha\left(1-\lambda_j\right)} \right) c_j$$

and profit is

$$\begin{aligned} \pi_j &= \left(p_j - c_j\right) q_j \\ &= \lambda_j \left(\frac{1-\alpha}{1-\alpha\lambda_j} \right). \end{aligned}$$

Let p_k be the price charged by a firm at the technological frontier when there are k leaders and let $\bar{p}_k$ be the price charged by a laggard when there are k leaders. For $k = 1, ..., n-1$ define

$$\hat{p}_k = \frac{p_k}{\bar{p}_k}.$$

Then we have:

$$\begin{aligned} \lambda_k &= \frac{1}{k + (n-k)\left(\hat{p}_k\right)^{\frac{\alpha}{1-\alpha}}}, \qquad k = 1, 2, .., n \\ \bar{\lambda}_k &= \frac{1}{k\left(\frac{1}{\hat{p}_k}\right)^{\frac{\alpha}{1-\alpha}} + n - k}, \qquad k = 1, 2, .., n-1 \\ \hat{p}_k &= \frac{1 + \frac{1-\alpha}{k+(n-k)\left(\hat{p}_k\right)^{\frac{\alpha}{1-\alpha}} - 1}}{1 + \frac{1-\alpha}{k\left(\frac{1}{\hat{p}_k}\right)^{\frac{\alpha}{1-\alpha}} + n - k - 1}} \left(\frac{1}{\gamma} \right), \qquad k = 1, 2, .., n-1 \\ \hat{p}_n &= 1 \end{aligned}$$

Inserted in the profit functions yields

$$\begin{aligned} \pi_k &= \frac{\lambda_k\left(1-\alpha\right)}{1-\alpha\lambda_k} = \frac{1-\alpha}{k + (n-k)\left(\hat{p}_k\right)^{\frac{\alpha}{1-\alpha}} - \alpha}, \qquad k = 1, 2, .., n \\ \bar{\pi}_k &= \frac{\bar{\lambda}_k\left(1-\alpha\right)}{1-\alpha\bar{\lambda}_k} = \frac{1-\alpha}{k\left(\frac{1}{\hat{p}_k}\right)^{\frac{\alpha}{1-\alpha}} + n - k - \alpha}, \qquad k = 1, 2, .., n-1. \end{aligned}$$