Competition with Personalized Pricing and Strategic Product Differentiation

Øystein Foros, Hans Jarle Kind og Mai Nguyen-Ones

Prosjektet har mottatt midler fra det alminnelige prisreguleringsfondet.
Competition with  
Personalized Pricing and Strategic Product Differentiation

Øystein Foros
NHH Norwegian School of Economics
oystein.foros@nhh.no

Hans Jarle Kind
NHH Norwegian School of Economics and CESifo
hans.kind@nhh.no

Mai Nguyen-Ones
NHH Norwegian School of Economics
mai.nguyen@nhh.no

Abstract: Consumers leave increasingly more digital footprints which improve firms’ ability to practice personalized pricing (first-degree price discrimination). We ask whether there exist strategic effects that reduce firms’ incentives to do so. To answer this question, we first note that it is optimal for a firm that price discriminates to set the purchasing price equal to marginal costs from consumers who buy from a rival. This is true independently of whether the rival has made any non-price commitments (e.g. strategic product differentiation). In contrast, if a firm uses uniform pricing, the rival has incentives to make strategic commitments that soften competition. Consequently, we find that firms might find it optimal to commit to uniform pricing to avoid being trapped in a highly competitive equilibrium. The key insight is that a firm’s incentives to undertake strategic price-softening behavior depend on the rival’s choice between uniform and personalized pricing, and not the firm’s own choice.

1We thank Arne Rogde Gramstad, Kenneth Fjell, Jarle Møen and seminar participants at Forskermøtet, FIBE and faculty seminars at NHH Norwegian School of Economics for useful discussions. Further, we thank Greg Shaffer for very helpful comments and suggestions.
1 Introduction

Personalized pricing (first-degree price discrimination) was once the prevailing pricing method in the retail sector. Indeed, prior to the mid-nineteenth century, sellers in the U.S. and Western Europe negotiated on prices with each individual customer (Phillips, 2012; Wallmeier, 2018). It was not until the 1860s that we saw a shift towards the present pricing standard, uniform pricing. The establishment of the first department stores initiated the shift. Personalized pricing requires detailed information both about purchasing prices for each single good and about individual consumers’ expected willingness to pay. It thus turned out to be an inefficient pricing method for department stores that offered a wide variety of products and served a large number of customers.\(^2\) Imposing one single fixed price on each good made the pricing task substantially less time consuming (Phillips, 2012, p.33), and by 1890 advertisements like "One Price for Every Man" and “One price to all” marked the uniform price policy as the new pricing norm (Phillips, 2012, p. 32; Resseguie, 1965, pp.302-303).\(^3\)

Today, personalized pricing is again on the agenda. Consumers use apps that are customized to collect individual data, and leave digital footprints on the Internet. In contrast to the early nineteenth century, sellers can directly learn about consumers’ willingness to pay.\(^4\) Moreover, Big Data and machine learning algorithms allow firms to come much

\(^2\) Clerks used to adopt a “price code” system where information about prices written on the price-tags was understandable only for the clerks and not for the customers (Phillips, 2013, p.30). Hence, when stores grew larger, not only was negotiation more time consuming, but keeping track of all the codes became more cumbersome as well.

\(^3\) Among pioneers was Alexander T. Stewart, who established a dry-goods store in New York in 1826. Stewart is often credited as being the first to use the one-price-to-all-principle in the United States. Britannica (2018) writes the following: "Instead of haggling over prices with each individual customer, Stewart set standard prices on all his goods, which was an innovation in his time." Macy’s announced its one-price policy in 1858 (Resseguie, 1965), and the same policy was applied by John Wanamaker in Philadelphia some years later. In Western Europe, some Parisian stores had one-price-to-all-ads already in the 1830s (Wallmeier, 2018; Resseguie, 1965; Phillips, 2012).

\(^4\) The high profile Facebook-Cambridge Analytica case illustrates that such information is not restricted to information directly collected from own consumers. Cambridge Analytica achieved access to private information from the counts of more than 50 million Facebook users. The firm’s tools could identify the personalities of American voters and influence their behavior, according to the New York Times (2018). Market players as well as politicians may use such information from intermediaries.
closer to applying personalized pricing than before, for instance by inducing a shift from third-degree (group based pricing) to first-degree price discrimination. Information costs are significantly reduced, and firms are often capable of practicing high-scale personalized pricing. In Varian’s (2010) terminology, "Instead of a ‘one size fits all’ model, the Web offers a ‘market of one’". This development may further give firms stronger incentives (and better abilities) to tailor their products to match individual preferences. By reducing the mismatch between basic product characteristics and what each single consumer prefers, the size of the market and the consumers’ willingness to pay for the good should increase.

This development raises the question of whether personalized pricing will again become the standard in retail markets. How do firms’ incentives and profitability from practicing personalized pricing compare to what we would observe if they practiced uniform pricing? Owing to textbook examples in ECO101, many relate personalized pricing to a monopolist seller who extracts all consumer surplus by charging each individual a price equal to her maximum willingness to pay for the good. Before the arrival of department stores 150 years ago, sellers were often local monopolists in their product lines (Jones, 1936, among others).\footnote{At that time, the general retail store in a region offering some product lines was often the only source of supply of goods which people could not produce themselves in their homes. Further, special stores offering one product line were rare and usually found only in large cities (Jones, 1936, p.134).} The advantage of using personalized pricing in such markets is well illustrated by the textbook example. However, in retail markets today, there are usually more than one seller; digitalization in itself increases the alternatives for consumers through online sales. If they use personalized pricing, firms might then end up competing intensively for each and every consumer (a “market of one”).\footnote{In their bestseller, written for a business audience, Shapiro and Varian (1998, pp. 40) gave a warning: "If your online travel agency knows that you are interested in deep-sea fishing, and it knows that deep-sea fishermen like yourself are often wealthy, it may well want to sell you a high-priced hotel package. On the other hand, if the travel agency knows that you like snorkeling, and snorkelers prefer budget travel, then they can offer you a budget package. In these examples, the provider can design a package that is optimized for your interests and charge you accordingly. But be careful about those premium prices for deep-sea fishermen: even wealthy deep-sea fishermen can change travel agencies."} As shown in the seminal paper by Thisse and Vives (1988), even though firms are better off if they all use uniform pricing, they could be trapped in a prisoner’s dilemma situation where each has incentives to unilaterally adopt personalized pricing.
There certainly exist examples of personalized pricing, for instance among hotel and airline agencies (see, e.g., Mohammed, 2017). However, most firms set a fixed price for each product, even when they have access to large amounts of consumer data. Hence, for the time being, a widespread shift to personalized pricing in retail markets seems to be absent. In the same vein, it is interesting to note that despite the information revolution and huge advances in for instance supply side management and computer assisted design, firms do not seem to match their products according to each consumer's preferences to such an extent as one might expect.

The continued prevalence of uniform pricing could partly be due to privacy concerns and resistance from consumers who dislike information gathering and personalized pricing (see Acquisti et al., 2016, for a comprehensive survey). Consumers might also consider personalized pricing ("haggling") as "unfair", and prefer to buy from firms that commit to "One Price for Every Man". Phillips (2012) argues that this effect can help explain the move from personalized to uniform pricing in the nineteenth century example above.

We abstract from these effects on the consumer side, and focus on strategic interactions between competing firms. In particular, we ask whether a firm by committing to uniform pricing might be able to prevent a rival from undertaking aggressive non-price decisions. More specifically, our research question is how a firm's incentives to reduce the level of mismatch cost (we consider other non-price commitments in an extension of the basic model) depends on its own and its competitor's choice of price policy (uniform pricing versus personalized pricing). We also ask whether endogenous non-price commitments change the prisoner's dilemma outcome from Thisse and Vives (1988) described above.

To approach these questions we consider competition between two firms located at each end of a Hotelling line. At stage 1, each firm can commit to using uniform pricing (price policy commitment). At stage 2, the firms simultaneously choose a firm-specific level of mismatch cost. At stage 3, the firms compete in prices. If a firm has not committed to uniform pricing at stage 1, it is free to choose between uniform pricing and personalized pricing at stage 3. Stages 1 and 3 of the game resemble Thisse and Vives (1988); however, they assume that the level of mismatch cost is exogenous. In contrast, we follow Ferreira

---

7 A recent example that literally fits into the spatial Hotelling framework is Staples who offered individual discounts based on the distance between the customers' location and the rival stores (Wall Street Journal, 2012).
and Thisse (1996) and let the mismatch cost be one of the firms’ choice variables.

In equilibrium, a firm that uses personalized pricing will set price equal to marginal cost towards all consumers who are buying from the rival. This is a robust result, see Thisse and Vives (1988) and Lederer and Hurter (1986), and is independent of the rival’s decisions on mismatch cost. In contrast, a firm that sets a uniform price will lower its price if the rival reduces its mismatch cost. This is true because the competitive pressure for the firm’s marginal consumer increases in the rival’s reduction of mismatch cost since the rival’s product becomes more attractive. Therefore, we show that a firm’s incentives to change its mismatch cost depend on the rival’s choice between uniform pricing and personalized pricing. A firm finds it optimal to reduce its own mismatch cost only if the rival uses personalized pricing; the optimal choice regarding the mismatch cost is independent of the firm’s own choice between price policies. Hence, a firm may choose to stick to uniform pricing in order to prevent the rival from reducing its mismatch cost and expanding its market. Personalized pricing comes at a cost because it triggers an aggressive response from the rival in tailoring its product to each consumer’s preferences, which is harmful for the other firm.

More generally, a rival using personalized pricing optimally sets price equal to marginal cost in the other firm’s market region, which means that the firm cannot affect the rival’s behavior towards these consumers by adjusting its non-price variable (such as mismatch cost or location). Hence, price discrimination by the rival, and the rival only, removes strategic effects of non-price commitments. To our knowledge, this has not yet been highlighted in the literature. In the spirit of Fudenberg and Tirole (1984) and Tirole (1988) we show that a firm’s choice of whether to commit to uniform pricing at stage 1 is a choice of whether to give the rival strategic incentives to undertake commitments in non-price variables.

The rest of the paper proceeds as follows. Section 2 reviews related literature. In Section 3 we set up the basic model with the standard assumptions in a Hotelling framework. Before solving the game we consider some general implications of personalized pricing on firms’ strategic incentives in non-price variables. We extend the model in three ways in Section 4 by considering a two-sided market, location incentives and by opening up for partial multi-homing by consumers. Lastly, Section 5 concludes.
2 Literature review

Recent developments in information gathering technologies make it possible for firms to collect more accurate information about consumers’ individual willingness to pay, and this increases firms’ abilities to practice personalized pricing (first-degree price discrimination). Therefore, personalized pricing is on the agenda as ever before. This is reflected in recent debates both in popular media (e.g. Forbes, 2014) and in academic literature (e.g. Esteves, 2010; Valletti and Wu, 2016; Prüfer and Schottmüller, 2017).

Our study is closely related to Thisse and Vives (1988), who consider a two-stage game where each of two Hotelling firms can commit to uniform pricing before they compete in prices. For a firm that does not commit to uniform pricing in the first stage, it is optimal to use personalized pricing in the second stage. Thisse and Vives (1988) show that a prisoner’s dilemma outcome emerges, where both firms in equilibrium use personalized pricing even though aggregate profit would have been higher if they both had committed to uniform pricing.\(^8\) We build on the framework developed by Thisse and Vives, but allow each firm to choose how closely it will match its good to individual consumer preferences; the poorer the match, the greater is the hedonic consumer price (the sum of monetary price and mismatch costs). The matching choice is made prior to the price competition stage, but after firms’ choice of whether to commit to uniform pricing. We show that once firms are able to make the matching choice, the prisoner’s dilemma outcome described above may cease to be an equilibrium: the firms may now choose to commit to uniform pricing.

Also Ferreira and Thisse (1996)\(^9\) open up for endogenous mismatch costs prior to the price competition stage. They consider a framework where two firms are located at each end of a Hotelling line, and show that each firm chooses to impose high own mismatch costs. This is similar to our finding under uniform pricing; going for high mismatch costs induces soft pricing behavior from the rival. Hendel and de Figueiredo (1997) assume a circular model instead of the Hotelling line, and arrive at the same qualitative result; in a setting with two firms, each of them chooses high mismatch costs in order to induce soft price competition. In contrast to us, neither Ferreira and Thisse (1996) nor Hendel and de

---

\(^8\)A similar outcome is reached a two-period framework in Fudenberg and Tirole (2000) and Esteves (2010).

\(^9\)Based on the firm-specific transportation cost framework from Launhardt (1885).
Figueiredo (1997) let firms choose between uniform and personalized pricing.\footnote{In von Ungern-Sternberg (1988) firms choose mismatch costs in a circular model. However, he assumes that mismatch costs and price are determined simultaneously. This implies that there is no strategic interdependence between these two choice variables.}

It is well established in the literature on personalized pricing that firms in equilibrium set price equal to marginal cost to its marginal consumer and to consumers served by the rival (Hurter and Lederer, 1985; Lederer and Hurter, 1986; Thisse and Vives, 1988; Bhaskar and To, 2004). We show that this has the interesting implication that, in the terminology of Fudenberg and Tirole (1984) and Tirole (1988), a firm’s choice of whether to commit to uniform pricing is also a choice of whether to give the rival strategic incentives to undertake non-price commitments. More precisely, if a firm uses personalized pricing, there will be no strategic effect of a rival’s choice of non-price commitment. This result hinges on the assumption that firms choose both price policy and a non-price variable prior to the competition stage. Previous studies assume either fixed price policy, such that both firms per definition use personalized pricing (Hurter and Lederer, 1985; Lederer and Hurter, 1986; Bhaskar and To, 2004) or no endogenous non-price commitments (Thisse and Vives, 1988). Therefore, our result that there is no strategic effect from a firm’s non-price commitment (e.g. mismatch costs) if the rival uses price discrimination is novel.

In an extension of the basic Hotelling model where firms are located at the extremes of the Hotelling line, we consider a firm that uses personalized pricing and show that its location incentives depend crucially on the pricing policy of the rival. The firm we consider perceives a rival that charges all consumers the same price (uniform pricing) as relatively soft. This indicates that it will locate closer to a rival that uses uniform pricing than to a rival that uses personalized pricing. However, as noted above, the strategic effect – which generates minimum differentiation in the standard Hotelling model – does not exist if the rival uses personalized pricing. We show that for this reason, the firm will nonetheless locate closer to a rival that uses personalized pricing than to a rival that uses uniform pricing. As a corollary, it follows that if both firms use personalized pricing, they will both have incentives to locate relatively close to each other. This result is consistent with Hurter and Lederer (1985), Lederer and Hurter (1986) and Bhaskar and To (2004), who show that if two firms compete with personalized pricing, they will choose interior locations on the Hotelling line (actually, they will choose the socially optimal locations). However, neither
of these studies consider the case where only one of the firms use personalized pricing. As such, their result on location is a special case of our general result. An important lesson from our analysis, is that it is not personalized pricing in itself that removes strategic effects of non-price commitments, it is personalized pricing by the rival that drives the result. As far as we know, this insight has not previously been acknowledged in the literature.

Our study also relates to the literature on product customization. Big data does not only put personalized pricing on the agenda, it also makes product customization a current topic as more information about consumer preferences is available. The mismatch cost in our model can be interpreted as product customization, where a firm can match its product better to consumers’ most preferred taste by decreasing the level of transportation cost. Dewan et al. (2000; 2003) and Bernhardt et al. (2007) consider costly customization. By contrast, we bypass any costs of customization in order to isolate the strategic effects on price. Syam et al. (2005) take a similar approach, though in a different context than ours. However, none of the above papers studies the choice of price policy in relation to product customization as we do.

3 The model set-up

We consider competition between two firms, \( i = 0, 1 \), located at the extremes of a Hotelling line with length 1. The location of firm \( i \) is given by \( x_i \), where \( x_i = 0 \) for firm 0 and \( x_i = 1 \) for firm 1. Consumer tastes are uniformly distributed along the line. Throughout, we assume that both firms are active (market sharing), and we consider both personalized and uniform pricing. Under personalized pricing (first-degree price discrimination) each consumer is given an individual price \( p_i(x) \), where \( x \) is the consumer’s location on the Hotelling line. Under uniform pricing all consumers pay the same price \( p_i(x) = p_i \), independently of location.

The consumer utility of buying from firm \( i \) for a consumer located at \( x \) can be written as

\[
    u_i(x) = v - m_i |x - x_i| - p_i(x). \tag{1}
\]

We assume that the parameter \( v > 0 \) is sufficiently large to ensure market coverage. The second term in (1) captures the idea that consumers will in general not find any of
the goods to be a perfect fit; the perceived mismatch costs associated with good $i$ for a consumer located at $x$ is $m_i \left| x - x_i \right|$, where $m_i > 0$. The smaller is $m_i$, the greater is the number of consumers who is willing to buy good $i$, other things equal. Put differently, decreasing $m_i$ enlarges the size of the market for firm $i$. This modelling of the mismatch costs is equivalent to the firm-specific transportation cost used by Ferreira and Thisse (1996)\textsuperscript{11}.

The location of the consumer who is indifferent between the offers from firm 0 and 1, denoted by $\tilde{x}$, is found by setting $u_0(\tilde{x}) = u_1(\tilde{x})$:

$$D_i = \frac{m_j + p_j(\tilde{x}) - p_i(\tilde{x})}{m_i + m_j}.$$  

(2)

Evidently, demand for good $i$ is decreasing in own mismatch costs, $\partial D_i / \partial m_i = -D_i / (m_i + m_j) < 0$, and increasing in the rival’s mismatch costs, $\partial D_i / \partial m_j = (1 - D_i) / (m_i + m_j) > 0$.

We analyze a three-stage game. At stage 1, each firm might commit to using uniform pricing towards the consumers (price policy commitment). Then, at stage 2, the firms simultaneously decide on mismatch levels. We assume that $m_i$ is bounded by $m_i \in [\underline{m}, \overline{m}]$. At stage 3, the firms compete in consumer prices. If firm $i$ has not made any commitment at stage 1, it is free to choose between using uniform pricing and personalized pricing at stage 3.

Each firm thus commits to uniform pricing if this is individually profitable. Such a commitment is consistent with the “one price to all” concept that was introduced by department stores 150 years ago when they through advertisement and money-back guarantees bound themselves to apply uniform pricing (Phillips, 2012). Without such a commitment, firms could be tempted to price according to what they expected each consumer to be willing to pay (personalized pricing).

Below, we first assume that one of the two firms, which we label firm $k$, has committed to uniform pricing, and analyze what effect this commitment might have on pricing and choice of mismatch costs. We consider both the case where the rival uses uniform pricing and where it uses personalized pricing. Then we perform the same analysis if firm $k$ has made no price policy commitment. Since the firms are intrinsically symmetric, we will, without loss of generality, let $k = 0$.

\textsuperscript{11}The modelling in Ferreira and Thisse (1996) builds on Launhardt (1885).
3.1 Preliminary insights: Implications of personalized pricing

Before we solve the game presented above, we show some general results on how personalized pricing affects firms’ incentives to undertake strategic commitments in non-price variables. A non-price variable can for instance be mismatch costs, as in our main model, or location on the Hotelling line (see section 4.2). Denote the level of the non-price variables by $n_0$ and $n_1$ (corresponding to $m_0$ and $m_1$ in the main model). We assume that firm 0 has committed to uniform pricing at stage 1. We maintain the assumption that the levels of the non-price variables are determined non-cooperatively at stage 2, and that these variables are observable when the firms compete in prices at stage 3.

First, consider the case where both firms have committed to uniform pricing. In general we cannot say whether prices are strategic complements or strategic substitutes, but for the sake of the argument (and without affecting the qualitative results below) we assume they are strategic complements. In either case the reduced form profit of firm 0 at stage 2 can be written as

$$\pi_0(n_0, n_1, p_0(n_0, n_1), p_1(n_0, n_1)). \quad (3)$$

The total derivative of (3) with respect to the non-price variable $n_0$ is

$$\frac{d\pi_0}{dn_0} = \frac{\partial \pi_0}{\partial n_0} + \left( \frac{\partial \pi_0}{\partial p_1} \right) \left( \frac{dp_1}{dn_0} \right). \quad (4)$$

where

$$\frac{dp_1}{dn_0} = \left( \frac{dp_1}{dp_0} \right) \left( \frac{dp_0}{dn_0} \right).$$

The first term on the right-hand side of (4) measures the change in firm 0’s profit when it increases $n_0$, holding the rival’s price $p_1$ fixed. This is the direct effect of changing $n_0$, and in equilibrium firm 0 would solve $\partial \pi_0/\partial n_0 = 0$ if $n_0$ was unobservable. Let $\hat{n}_0$ denote the solution to $\partial \pi_0/\partial n_0 = 0$.

Since we have assumed that $n_0$ is observable prior to the price decision in stage 3, $p_1$ is a function of $n_0$. Firm 0 thus has incentives to strategically affect the price charged by the rival through the level of the non-price variable $n_0$ (in normal cases $\partial \pi_0/\partial p_1 > 0$). This effect is captured by the second term on the right-hand side of (4). Suppose that
Given the assumption that prices are strategic complements \( dp_1/dp_0 > 0 \), it follows that firm 0 will then commit to \( n_0 > \hat{n}_0 \) because this induces the rival to increase its price too. In the terminology of Fudenberg and Tirole (1984), firm 0 chooses a "fat cat strategy"; it "overinvests" in the non-price variable to appear soft (it charges a higher price). In contrast, if the "investment" makes firm 0 tough (i.e., \( dp_0/dn_0 < 0 \)), it commits to a lower value of the non-price variable \( (n_0 < \hat{n}_0) \) in order to make the rival set a relatively high price. This corresponds to a "puppy dog strategy" in the terminology of Fudenberg and Tirole.

Now, consider instead the case where firm 1 has not made a commitment to uniform pricing at stage 1. For now, we assume that firm 0 knows firm 1 has incentives to use personalized pricing at stage 3 in this case (we will later verify that this holds). As shown in the seminal contributions by Thisse and Vives (1988) and Lederer and Hurter (1986), a firm using personalized pricing will charge an individual price equal to the marginal cost to the "last" consumer it serves as well as all consumers served by the rival. Hence, in stage 3 firm 1 offers \( p_1(\bar{x}) = c \) towards all consumers served by firm 0. This price decision is independent of the non-price commitments made in stage 2 \( (n_0 \text{ and } n_1) \). Firm 0’s profit is then given by

\[
\pi_0(n_0, n_1, p_0(n_0, n_1), p_1(\bar{x})).
\] (5)

The total derivative of (5) is

\[
\frac{d\pi_0}{dn_0} = \frac{\partial \pi_0}{\partial n_0} + \frac{\partial \pi_0}{\partial p_1(\bar{x})} \cdot \frac{dp_1(\bar{x})}{dn_0},
\]

where

\[
\frac{dp_1(\bar{x})}{dn_0} = 0.
\]

Hence, the strategic effect is eliminated: When firm 1 uses personalized pricing, firm 0 cannot strategically affect firm 1’s pricing behaviour, \( p_1(\bar{x}) = c \). Neither can firm 0 affect \( p_1(\bar{x}) = c \) through its choice of whether to commit to uniform pricing at stage 1.

Therefore, we have the following general result: If a firm faces a rival which uses personalized pricing, non-price commitments have no strategic effect. We can state:
**Proposition 1:** Suppose that firm 1 uses personalized pricing. Then, there is no strategic effect neither from firm 0’s possible commitment to uniform pricing nor from its commitment to the non-price variable \(n_0\).

Proposition 1 implies that the choice of whether to commit to uniform pricing or not at stage 1 can be seen as a choice of whether to eliminate the rival’s strategic incentives to undertake non-price commitments at stage 2. Put differently, a firm may commit to uniform pricing if it is profitable that the rival undertakes a strategic commitment at stage 2. In contrast, if it is profitable that the rival does not undertake a strategic commitment at stage 2, the firm may choose not to commit to uniform pricing.

It follows from Thisse and Vives (1988) and Lederer and Hurter (1986) that a firm using personalized pricing offers an individual price equal to marginal cost to all consumers served by the rival. However, Thisse and Vives (1988) do not consider endogenous non-price commitments (they do not have stage 2 in our model), while Lederer and Hurter (1986) assume that both firms use personalized pricing (they do not consider stage 1 in our model). Hence, none of them consider this general implication.

### 3.2 Firm 0 has committed to uniform pricing

#### 3.2.1 Pricing (stage 3)

We now return to the specific model set-up in order to solve the corresponding game. Using backward induction, we start with the firms’ pricing decisions (stage 3). At this stage the firms’ product characteristics (mismatch costs) and price policies (whether they have committed to uniform pricing) are predetermined.

If firm 0 at stage 1 has committed to uniform pricing, it will solve the following maximization problem:

\[
\max_{p_0} \pi_0^{UP-R} = (p_0 - c)D_0^{UP-R}, \text{ where } R \in \{UP, PP\}. \tag{6}
\]

Throughout, the first part of the superscript indicates the firm’s own price strategy (uniform pricing, abbreviated to \(UP\), in this case), and the second part indicates the rival’s price strategy (where \(R\) is \(UP\) or \(PP\), where the latter stands for personalized pricing).

Suppose first that also firm 1 has committed to uniform pricing. Setting \(p_i(x) = p_i\) and \(p_j(x) = p_j\) into equation (2) it follows that perceived demand for firm \(i = 0, 1\) equals:
By solving (6) we now find that prices are strategic complements, and that the reaction functions are given by

\[ p_i(p_j) = \frac{c + p_j}{2} + \frac{m_j}{2}. \]  

A higher value of \( m_j \) means that the competitive pressure for firm \( i \)'s marginal consumers falls. This explains why \( \frac{\partial p_i(p_j)}{\partial m_j} > 0 \). In contrast, we see that \( \frac{\partial p_i(p_j)}{\partial m_i} = 0 \); firm \( i \)'s optimal price does not depend directly on its own choice of mismatch costs. The reason for this is that a higher value of \( m_i \) reduces the number of consumers who prefers good \( i \), but does not affect the optimal price towards its remaining consumers, all else equal. However, since an increase in \( m_i \) increases the rival’s price, we nonetheless find that each firm’s (potential) equilibrium price is increasing both in its own and the rival’s mismatch costs, albeit most in the latter. More precisely, solving (8) for the two firms’ prices simultaneously, we have

\[ p_{UP-UP}^i = c + \frac{m_i + 2m_j}{3}, \]  

proving that \( \frac{\partial p_{UP-UP}^i}{\partial m_j} > \frac{\partial p_{UP-UP}^i}{\partial m_i} > 0 \).

Inserting for (7) and (9) into (6) yields

\[ \pi_{UP-UP}^i = \frac{(m_i + 2m_j)^2}{9(m_i + m_j)}, \]  

from which it follows that \( \frac{\partial \pi_{UP-UP}^i}{\partial m_j} > \frac{\partial \pi_{UP-UP}^i}{\partial m_i} > 0 \). Since higher mismatch cost softens competition when both firms use uniform pricing, it leads to higher profits.

Suppose next that only firm 0 has committed to uniform pricing. Firm 1 is then free to choose between uniform pricing and personalized pricing at the stage 3, but it will clearly select the latter. The reason for this is that with personalized pricing, it can charge a price from each consumer which is infinitesimally lower than that of firm 0 and become these consumers’ preferred supplier (and this will be the optimal pricing strategy towards all consumers who thereby generates a non-negative profit). No other price schedule can possibly yield a higher profit for firm 1. Following Thisse and Vives (1988), we thus assume
that when only firm 0 has made a price policy commitment, it will act as a Stackelberg leader at stage 3. Inserting \( p_1^{PP}(\bar{x}) = c \) into (2), it follows that firm 0’s demand becomes

\[
\bar{x} = D_0^{UP-PP} = \frac{m_1 - (p_0 - c)}{m_0 + m_1}.
\]

By solving the maximization problem in (6) we then find

\[
p_0^{UP-PP} = c + \frac{m_1}{2}.
\] (11)

Equation (11) is firm 0’s equilibrium price as well as its reaction function. The latter follows because the rival always charges a price equal to marginal costs for its last consumer and for all consumers served by firm 0 (so that \( p_1(x) = c \) for \( x \in [0, \bar{x}] \)).

Profit of firm 0 can now be written as

\[
\pi_0^{UP-PP} = \frac{m_1^2}{4(m_0 + m_1)}.
\] (12)

Firm 1 sells to all consumers in the interval \([\bar{x}, 1]\), and these consumers are charged prices which ensure that \( u_1(x) \geq u_0(x) \). In equilibrium this constraint is binding, and from equation (1) we find that \( p_1(x) = c + \frac{m_1}{2} + m_0x - m_1(1 - x) \) for \( x \in [\bar{x}, 1] \). Profit for firm 1 thus equals

\[
\pi_1^{PP-UP} = \int_{\bar{x}}^{1} (p_1(x) - c) \, dx = \frac{(2m_0 + m_1)^2}{8(m_0 + m_1)}.
\] (13)

3.2.2 Choice of mismatch costs (stage 2)

Let us now turn to firm 0’s choice of mismatch costs (stage 2). With no effect on our qualitative results, we assume that the firm can costlessly choose any mismatch level it wants within the boundaries \([m, \bar{m}]\).

By assumption, firm 0 has committed to uniform pricing. If the rival has made the same commitment (recall that it will not use uniform pricing at stage 3 unless it has committed to do so), we know from equations (9) and (10) that equilibrium prices and profits are increasing in each firm’s level of mismatch costs. It thus follows that firm 0 will set \( m_0 = \bar{m} \) (and firm 1 will likewise set \( m_1 = \bar{m} \)).

\[12\] If firms set prices simultaneously when one of them has committed to uniform pricing and the other uses personalized pricing, then we must solve for mixed strategies. See Thisse and Vives (1988, 1992).
In the terminology of Fudenberg and Tirole (1984) and Tirole (1988), cf. section 3.1, firm 0 uses a puppy dog strategy if the rival uses uniform pricing: it "underprovides" reductions in the mismatch level on its own good in order to induce a more soft response from the rival. This is similar to the findings in Ferreira and Thisse (1996), and is related to findings in the literature on strategic obfuscation (obfuscation complicates or prevents consumers from gathering price information). Ellison & Wolitzky (2012) show that firms may unilaterally choose to raise consumers’ search costs. This may be considered as analogue to raising their own mismatch costs.

In contrast, if the rival uses personalized pricing, we know from Proposition 1 that a change in firm 0’s mismatch costs does not affect firm 1’s pricing behavior towards its marginal consumer or any of the consumers served by firm 0; it always sets \( p_1^{PP}(x) \big|_{x \leq \hat{x}} = c \). Consequently, as the strategic effect is eliminated firm 0 needs not worry about any aggressive response from the rival if it reduces the perceived mismatch costs associated with the good it offers. Since a reduction in own mismatch costs raises its market share \( (\partial D_0^{UP-PP}/\partial m_0 < 0) \), firm 0 thus maximizes profit by setting \( m_0 = m \). Formally, this follows because equation (12) implies:

\[
\frac{\partial \pi_0^{UP-PP}}{\partial m_0} = -\frac{m_1^2}{4(m_0 + m_1)^2} < 0
\]

To summarize the results so far:

**Lemma 1:** Suppose that firm 0 has committed to uniform pricing, and that the rival

(a) uses uniform pricing. Then firm 0 chooses to maximize mismatch costs associated with its own good (sets \( m_0^{UP-UP} = \overline{m} \)).

(b) uses personalized pricing. Then firm 0 chooses to minimize mismatch costs associated with its own good (sets \( m_0^{UP-PP} = m \)).

### 3.3 Firm 0 has not committed to uniform pricing

#### 3.3.1 Pricing (stage 3)

Suppose that firm 1 has committed to uniform pricing, while firm 0 has made no commitment. Then we know from the analysis above that firm 0 will use personalized pricing. Due to the intrinsic symmetry of the firms, we can switch subscripts in equation (13) and
deduce that the profit level of firm 0 now equals

\[ \pi_0^{PP-UP} = \int_0^{\hat{x}} (p_0(x) - c) \, dx = \frac{(m_0 + 2m_1)^2}{8(m_0 + m_1)}. \]  

From equations (11) and (12) it likewise follows that

\[ p_1^{UP-PP} = c + \frac{m_0}{2} \]  
\[ \pi_1^{UP-PP} = \frac{m_0^2}{4(m_0 + m_1)}. \]

Suppose instead that neither of the firms have committed to uniform pricing. In this case both firms will use personalized pricing.\(^\text{13}\) Each of them will consequently set price equal to marginal cost for its last consumer \((x = \hat{x})\) and for all consumers served by the rival (Thisse and Vives, 1988). Hence, inserting \(p_0^{PP}(\hat{x}) = p_1^{PP}(\hat{x}) = c\) into (2) yields

\[ \hat{x} = D_0^{PP-PP} = \frac{m_1}{m_0 + m_1}. \]  

Equivalently, \(D_1^{PP-PP} = 1 - \hat{x} = \frac{m_0}{m_0 + m_1}. \)\(^\text{14}\)

Profit to firm \(i\) is then\(^\text{15}\)

\[ \pi_i^{PP-PP} = \frac{m_j^2}{2(m_i + m_j)}. \]

### 3.3.2 Choice of mismatch costs (stage 2)

Now, consider firm 0’s incentives to reduce mismatch costs when it uses personalized pricing. Assume first that firm 1 uses uniform pricing. The discussion above then indicates that firm 0 will choose high mismatch costs, because this makes firm 1 soft. This is confirmed by differentiating equation (14) with respect to \(m_0:\)

\(^\text{13}\)In equation (18) below we find that \(\pi_i^{PP-PP} = \frac{m_i^2}{2(m_i + m_j)}.\) Since \(\pi_i^{PP-PP} - \pi_i^{UP-PP} = \frac{m_i^2}{2(m_i + m_j)} - \frac{m_i^2}{4(m_i + m_j)} > 0\) and \(\pi_i^{PP-UP} - \pi_i^{UP-UP} = \frac{(2m_j + m_i)^2}{8(m_0 + m_1)} - \frac{(2m_j + m_i)^2}{8(m_0 + m_1)} = \frac{1}{16} \left( \frac{2m_j + m_i}{m_0 + m_1} - 1 \right) > 0\) it follows that firm \(i\) will use personalized pricing whatever the price policy of the rival. Thus, it is a dominant strategy at stage 3 to choose personalized pricing for a firm that has not made any other commitment.

\(^\text{14}\)It is straightforward to show that if firm 0 uses personalized pricing it will sell less if the rival uses personalized pricing than if the rival uses uniform pricing \((D_0^{PP-PP} < D_0^{PP-UP}).\) The reason for this is that the rival sets a lower price towards its marginal consumer in the former case \(p_0^{PP}(\hat{x}) = c < p_0^{UP-PP} = c + m_0/2).\)

\(^\text{15}\)We have \(\pi_0^{PP-PP} = \int_0^{\hat{x}} [p_0(x) - c] \, dx = \frac{m_0^2}{2(m_0 + m_1)}\) and \(\pi_1^{PP-PP} = \int_{\hat{x}} \pi_1(x) - c \, dx = \frac{m_0^2}{2(m_0 + m_1)}.\)
\[
\frac{\partial \pi_{0}^{PP-UP}}{\partial m_{0}} = \frac{(m_{0} + 2m_{1})m_{0}}{8(m_{0} + m_{1})^{2}} > 0.
\]

If firm 1 instead uses personalized pricing, it sets \(p_{1}^{PP}(x) = c\) towards its marginal consumer. We again know from Proposition 1 that firm 0 then is unable to make its rival softer through choosing high mismatch costs. It is therefore unambiguously beneficial for firm 0 to reduce mismatch costs, because this will increase the size of its market. Formally, from equation (18), we have

\[
\frac{\partial \pi_{0}^{PP-PP}}{\partial m_{0}} = -\frac{m_{1}^{2}}{2(m_{0} + m_{1})^{2}} < 0.
\]

We can state:

**Lemma 2:** Suppose that firm 0 uses personalized pricing, and that the rival
(a) uses uniform pricing. Then firm 0 chooses to maximize mismatch costs associated with its own good (sets \(m_{0}^{PP-UP} = \bar{m}\)).

(b) uses personalized pricing. Then firm 0 chooses to minimize mismatch costs associated with its own good (sets \(m_{0}^{PP-PP} = \bar{m}\)).

Lemma 2 resembles Lemma 1. Each firm takes into account the fact that if the rival uses uniform pricing, then a reduction of its own mismatch costs triggers an aggressive price response from the rival. If the rival uses personalized pricing, on the other hand, a firm which decreases its mismatch costs will observe higher sales without having to reduce its price. We thus have the following striking result, which is a main lesson from the current model:

**Proposition 2:** Firm \(i\)'s incentives to reduce the mismatch costs of its product is independent of whether it uses uniform prices or not. It chooses to reduce mismatch costs if and only if the rival uses personalized pricing.

Proposition 2 highlights the fact that choosing personalized pricing comes at a cost; it gives your rival incentives to tailor its good to each consumer’s preferences (reduce mismatch costs). In the next section we will consider whether this effect may induce firms not to choose personalized pricing.

Note that even though a reduction in mismatch costs is individually profitable, the firms would be better off if they could make a (joint) commitment to abstain from it. To see
this, assume \( m_1 = m_2 = m \). Equation (18) is then simplified to \( \pi_i^{PP-PP} \big|_{m_i=m_j=m} = m/4 \), which is strictly increasing in \( m \).

### 3.4 The choice of personalized pricing

Using the results that firm \( i \) sets \( m_i = \bar{m} \) (minimum mismatch costs) if the rival uses personalized pricing and \( m_i = \overline{m} \) if the rival uses uniform pricing, we can apply equations (10) and (18) to express profit if both firms use either uniform pricing or personalized pricing as respectively

\[
\pi_i^{UP-UP} = \frac{\bar{m}}{2} \quad \text{and} \quad \pi_i^{PP-PP} = \frac{m}{4}. \tag{19}
\]

If one and only one of the firms has committed to uniform pricing, we likewise find from equations (12) and (13) that

\[
\pi_i^{PP-UP} = \frac{(\overline{m} + 2m)^2}{8(m + \overline{m})} \quad \text{and} \quad \pi_i^{UP-PP} = \frac{m^2}{4(m + \overline{m})}. \tag{20}
\]

Let \( \alpha \equiv \frac{\overline{m}}{m} \geq 1 \) define the ratio between maximum and minimum mismatch costs, and suppose that firm \( j \) has committed to uniform pricing. Should firm \( i \) do the same? If it does, firm \( j \) will choose high mismatch costs (soft behavior). Equations (19) and (20) yield

\[
\pi_i^{UP-UP} - \pi_i^{PP-UP} = \frac{3\alpha^2 - 4}{8(1+\alpha)}m < 0 \quad \text{if} \quad \alpha < \alpha_{crit} = \sqrt{4/3} \approx 1.547. \tag{21}
\]

Thus, it is not a Nash equilibrium for both firms to choose uniform pricing if the ratio between maximum and minimum mismatch costs is below a critical value, \( \alpha < \alpha_{crit} \). The reason for this is that the gain from committing to uniform pricing and making the rival soft is then low compared to the gain from charging each consumer according to her willingness to pay for the good (personalized pricing). On the other hand, if \( \alpha > \alpha_{crit} \), we see that \( \pi_i^{UP-UP} - \pi_i^{PP-UP} > 0 \). Then, neither firm will regret committing to uniform pricing, because each of them has much to gain from having a soft rival.

What should firm \( i \) do if the rival has not committed to uniform pricing (which implies that it will use personalized pricing)? Using equations (19) and (20) we find

\[
\pi_i^{UP-PP} - \pi_i^{PP-PP} = \frac{\alpha(\alpha - 1) - 1}{4(\alpha + 1)}m > 0 \quad \text{if} \quad \alpha > \alpha^{crit} = \frac{1}{2}\sqrt{5} + \frac{1}{2} \approx 1.618. \tag{22}
\]

Hence, it is profitable for firm \( i \) to commit to uniform pricing even if the rival uses personalized pricing if \( \alpha > \alpha^{crit} \). Again, the intuition is that the larger is the ratio between
maximum and minimum mismatch costs, the more valuable it is to commit to uniform pricing in order to make the rival soft. The reason why $\alpha^{crit} > \alpha_{crit}$ is that the loss in market share from using uniform pricing is greater when the rival chooses personalized pricing than when it uses uniform pricing.

Inspection of (21) and (22) reveals that there does not exist any equilibrium where one firm commits to uniform pricing and the other does not\textsuperscript{16}, so we can state

**Proposition 3:** Equilibrium constellations:

(i) If $\alpha < \alpha_{crit}$, there is a unique equilibrium where both firms choose personalized pricing.

(ii) If $\alpha > \alpha^{crit}$, there is a unique equilibrium where both firms choose uniform pricing.

(iii) If $\alpha_{crit} \leq \alpha < \alpha^{crit}$, there are multiple equilibria, where both firms choose personalized pricing or both firms choose uniform pricing.

In sharp contrast to Thisse and Vives (1988), we thus find that it is not necessarily true that firms unambiguously will choose personalized pricing (which would be a prisoner’s dilemma). On the contrary, once we open up for endogenous mismatch costs, personalized pricing might not even constitute a Nash equilibrium. This is true if the span between the lowest and the highest level of mismatch costs is sufficiently large. The threat that the rival will tailor its product as closely as possible to each consumer’s preferences may discipline firms and induce them to stick to uniform pricing.

4 Extensions

4.1 The mixed blessing of accessing a two-sided market

In this section, we modify the model to consider a two-sided market. One example of firms or platforms in this context is newspapers, which attract readers as well as advertisers. Another example is search engines, serving users and advertisers. Suppose firms have two sources of revenue; they charge users for their consumption, as in the main model. In addition, they charge advertisers for providing them with the users’ attention. To keep

\textsuperscript{16}This might change if the firms are ex ante asymmetric, e.g. with respect to initial data accumulation.
the framework simple, we assume that consumers are indifferent to ad levels. Hence, their utility is unaffected by the advertisement side of the market.

If firm \( i \) uses uniform pricing in the user market, it charges each user a subscription fee \( p_i \). Further, as in Anderson et al. (2017a), we assume that the firm earns \( b \) per user in the advertising market. Its profit is therefore \( \pi_{UP-R}^i = (p_i + b - c)D_i \).

First, suppose both firms use uniform pricing in the user market. Solving \( \frac{\partial \pi_{UP.UP}^i}{\partial p_i} = 0 \), \( i = 1, 2 \), we find

\[
p_i = c - b + \frac{m_i + 2m_j}{3}.
\]

Compared to the main model, the user price is in this case \( b \) units lower. This is because the possibility of selling the users’ attention to advertisers intensifies firm rivalry to such an extent that they compete away advertising revenue. This so-called see-saw effect is well-known from the media economics literature (see e.g. Armstrong, 2006). Total profit for firm \( i \) is thus equal to

\[
\pi_{UP-UP}^i = \frac{(m_i + 2m_j)^2}{9 (m_i + m_j)},
\]

which is the same expression as in the main model, cf. equation (10).

Assume instead that firm \( i \) uses personalized pricing in the user market. Since this requires relatively disaggregated market data, it is reasonable to assume that the firm has acquired (weakly) more information about each individual user than it would under uniform pricing. Such individualized information could be valuable for the firm when it approaches the advertising market. To capture this, assume that firm \( i \) which uses personalized pricing can charge an ad premium \( \delta \geq 0 \) for each user. The profit level of firm \( i \) is then \( \pi_{PP-R}^i = (p_i(x) + b + \delta - c)D_i \).

In order to see the implications of the ad price premium, suppose that firm 1 uses personalized pricing, while firm 0 has committed to uniform pricing. A user located in \( x \) is now worth \( p_1(x) + b + \delta - c \) to firm 1, which is \( \delta \) units more than if it instead used uniform pricing. This hurts firm 0 in two ways. First, demand for good 0 falls, since the rival finds it profitable to capture more users with personalized pricing than with uniform pricing. More precisely, the location of firm 1’s marginal consumer is now implicitly given by \( p_{1PP}^* = c - b - \delta \), where \( \tilde{x} \) evidently is decreasing in \( \delta \). Second, since firm 1 is now willing to offer its good at a price equal to \( c - b - \delta \) to all consumers served by the rival, the perceived willingness to pay for good 0 falls (firm 0’s demand curve shifts \( \delta \) units
downward). Firm 0’s profit maximizing price is therefore strictly decreasing in \( \delta \). Formally, inserting for \( p_1^{PP}(\tilde{x}) \) into (2) and maximizing \( \pi_0 = (p_0 + b - c) D_0^{UP-PP} \) with respect to \( p_0 \) yields

\[
\tilde{x} = D_0^{UP-PP} = \frac{m_1 - \delta}{2(m_0 + m_1)} \quad \text{and} \quad p_0^{UP-PP} = c - b + \frac{m_1 - \delta}{2}.
\] (23)

Note that firm 0 will have positive sales only if \( m_1 > \delta \). To ensure that this is always the case, we assume that \( \overline{m} > \delta \). From (23) we then find that the profit level of firm 0 equals

\[
\pi_0^{UP-PP} = \frac{(m_1 - \delta)^2}{4(m_0 + m_1)}, \quad \frac{\partial \pi_0^{UP-PP}}{\partial \delta} = -\frac{1}{2} \frac{m_1 - \delta}{m_0 + m_1} < 0.
\]

We derive firm 1’s optimal price from equation (1) by setting \( u_0 = u_1 \). This yields \( p_1(x) = c - b + \frac{m_1 - \delta}{2} + m_0 \tilde{x} - m_1 (1 - \tilde{x}) \). The fact that firm 0’s optimal price falls when firm 1 uses personalized pricing forces firm 1 to reduce its price even towards consumers in its own turf. However, since firm 1 sells more and makes a higher profit per user the greater is \( \delta \), its profit level is nonetheless unambiguously increasing in \( \delta \):

\[
\pi_1^{PP-UP} = \int_{\tilde{x}}^{1} ((p_1(x) + b + \delta - c)) \, dx = \frac{(2m_0 + m_1 + \delta)^2}{8(m_0 + m_1)}.
\] (24)

Finally, it is straightforward to show that if both firms use personalized pricing, the see-saw effect once again implies that they compete away advertising revenue. Their profit level is thus the same as they would have been in the one-sided market, cf. equation (18):

\[
\pi_i^{PP-PP} = \frac{m_i^2}{2(m_i + m_j)}.
\]

As in the main model, each firm chooses to maximize mismatch costs (\( \overline{m} \)) if the rival uses uniform pricing and minimize mismatch costs (\( m \)) if the rival uses personalized pricing. Profits can then be expressed as

\[
\pi_i^{UP-UP} = \frac{\overline{m}}{2}, \quad \pi_i^{PP-PP} = \frac{m}{4}
\]

\[
\pi_i^{UP-PP} = \frac{(\overline{m} - \delta)^2}{4(\overline{m} + m)}, \quad \pi_i^{PP-UP} = \frac{(2\overline{m} + m + \delta)^2}{8(\overline{m} + m)}.
\]

From (25) it follows that \( d(\pi_i^{UP-UP} - \pi_i^{PP-UP}) / d\delta < 0 \) and \( d(\pi_i^{UP-PP} - \pi_i^{PP-PP}) / d\delta < 0 \). This implies that firm \( i \) is more incentivized to use personalized pricing the greater \( \delta \) is. We can thus state:
Proposition 4: Suppose that each firm has more individual reader data if it uses personalized pricing than if it uses uniform pricing in the user market. Suppose further that this generates a premium in the advertising market. The greater is the premium, the greater are each firm’s individual incentives to use personalized pricing, which can lead them to end up in the low-profit equilibrium with personalized pricing.

Profits are the same under a two-sided market and a one-sided market when firms use the same price policy due to the see-saw effect. However, the premium makes firms more incentivized to unilaterally adopt personalized pricing in a two-sided market compared to a one-sided market. Therefore, firms might prefer a one-sided market if a two-sided market induces switching to personalized pricing.

4.2 Location incentives

In this section, we extend the model to consider location incentives. In relation to section 3.1, location is a non-price variable. As such, it is interesting to examine the insights from Proposition 1 on firms’ location.

We assume that firm 0 uses personalized pricing and ask how its location incentives depend on firm 1’s choice between uniform pricing and personalized pricing. A full-fledged location analysis will not be carried out. Instead, we take firm 1’s location as given and examine firm 0’s location choice. We further set $m_0 = m_1 = m$ in order to highlight the effects on location.

First, we find the profit expression for firm 0. Let firm 1 be located at $x_1 \in (\frac{1}{2}, 1]$ and firm 0 at some point $x_0$ to the left of firm 1, as shown in Figure 1.

The net utility of buying good 0 for a consumer located (weakly) to the right of $x_0$ is $u_0^{x \geq x_0}(x) = v - m(x - x_0) - p_0(x)$, while the net utility of buying good 1 for a consumer to the left of $x_1$ equals $u_1^{x \leq x_1}(x) = v - m(x_1 - x) - p_1(x)$. Using the fact that firm 0 charges $p_0^{pp}(x) = c$ from the consumer who is indifferent between good 0 and good 1, we find from

\[^{17}\text{We now go back to the one-sided market context.}\]
\[^{18}\text{Technically, the way we have modelled mismatch costs corresponds to linear transportation costs. It is well known that this is unsuited for analyzing endogenous location when firms use uniform pricing (see e.g. d’Aspremont et al., 1979).}\]
\( u_{0}^{x \geq x_{0}}(\bar{x}) = u_{1}^{x \leq x_{1}}(\bar{x}) \) the demand facing firm 0

\[ D_{0} = \bar{x} = \frac{x_{0} + x_{1}}{2} + \frac{p_{1}(x) - c}{2m}. \]

Firm 0 maximizes profit by choosing \( p_{0}(x) \) such that \( u_{0}^{x \geq x_{0}} = u_{1}^{x \leq x_{1}} \) for all consumers between \( x_{0} \) and \( \bar{x} \). This means that

\[ p_{0}^{x \in [x_{0}, \bar{x}]}(x) = p_{1}(x) + m(x_{0} - x) - m(x - x_{1}) \text{ for } x \in [x_{0}, \bar{x}]. \] \( \text{(26)} \)

For consumers located between 0 and \( x_{0} \) the net utility of buying good 0 is \( u_{0}^{x < x_{0}} = v - m(x_{0} - x) - p_{0}(x) \). In this area firm 0 optimally sets \( p_{0}(x) \) such that \( u_{0}^{x < x_{0}} = u_{1}^{x \leq x_{1}} \), yielding prices

\[ p_{0}^{x \in [0, x_{0}]}(x) = p_{1}(x) - m(x_{0} - x) - m(x - x_{1}) \text{ for } x \in [0, x_{0}]. \]

Profit for firm 0 is thus

\[ \pi_{0}^{PP-R} = \int_{0}^{x_{0}} \left( p_{0}^{x \in [0, x_{0}]}(x) - c \right) dx + \int_{x_{0}}^{\bar{x}} \left( p_{0}^{x \in [x_{0}, \bar{x}]}(x) - c \right) dx, \]

which can be rewritten as

\[ \pi_{0}^{PP-R} = x_{0} \left( -c + p_{1}(x) + m(x_{1} - x_{0}) \right) + \frac{(-c + p_{1}(x) + m(x_{1} - x_{0}))^{2}}{4m}. \] \( \text{(27)} \)

Suppose that firms compete in prices at stage 2, and that firm 0 chooses location at stage 1 (recall that we take firm 1’s location as given). We solve the game through backward induction. After solving the the second stage problem, the first-order condition for stage 1 is given by (cf. section 3.1)

\[ \frac{d\pi_{0}}{dx_{0}} = \frac{\partial \pi_{0}}{\partial x_{0}} + \frac{\partial \pi_{0}}{\partial p_{1}} \frac{dp_{1}}{dx_{0}} = 0. \] \( \text{(28)} \)

From equation (27) we obtain

\[ \frac{\partial \pi_{0}^{PP-R}}{\partial p_{1}} = \frac{-c + p_{1}(x) + m(x_{0} + x_{1})}{2m} > 0, \]

which is unambiguously positive since \( p_{1}(x) \geq c \). We can now examine how the first-order condition of firm 0’s location problem depends on firm 1’s choice between uniform and personalized pricing.
If firm 1 uses personalized pricing, it will offer its good at a price equal to marginal cost for consumers located in \( x \in [x_0, \bar{x}] \). Inserting \( p_1^{PP}(x) = c \) in equation (27) we then find
\[
\pi_0^{PP-PP} = x_0 m (x_1 - x_0) + \frac{m (x_1 - x_0)^2}{4}.
\tag{30}
\]
Since \( p_1^{PP}(x) = c \) in \( x \in [0, \bar{x}] \), firm 0 cannot affect the price that firm 1 charges consumers in this area, that is, \( \frac{dp_1}{dx_0} = 0 \). Therefore, the total derivative in equation (28) reduces to \( \frac{d\pi_0}{dx_0} = \frac{\partial \pi_0}{\partial x_0} \). This resembles Proposition 1; only the market expansion (direct) effect of firm 0’s choice of location on profit remains when firm 1 uses personalized pricing. From (30) we find
\[
\frac{d\pi_0^{PP-PP}}{dx_0} = \frac{\partial \pi_0^{PP-PP}}{\partial x_0} = m (x_1 - 2x_0) - \frac{1}{2} m (x_1 - x_0) = \frac{m (x_1 - 3x_0)}{2}.
\]
Consequently, solving (28) for firm 0’s location yields \( x_0^{PP-PP} = \frac{1}{3} x_1 \).\(^{19}\)

If instead firm 1 uses uniform pricing, it solves \( p_1 = \arg \max \pi_1^{UP-PP} \), where \( \pi_1^{UP-PP} = (p_1 - c) (1 - D_0) \). This gives the price
\[
p_1 = \frac{2 (c + m) - m (x_0 + x_1)}{2}.
\tag{31}
\]
Firm 0 faces relatively soft (potential) competition when firm 1 uses uniform pricing. Other things equal, the firm will therefore expand demand more if it locates closer to a rival using uniform pricing compared to a rival using personalized pricing. Therefore, we should expect firm 0 to locate closer to its rival when the rival uses uniform pricing. To confirm this, note that
\[
\frac{\partial \pi_0^{PP-UP}}{\partial x_0} = -c + p_1 - 3mx_0 + mx_1 = \frac{m (2 - 7x_0 + x_1)}{4}.
\]
Since
\[
\frac{\partial \pi_0^{PP-UP}}{\partial x_0} - \frac{\partial \pi_0^{PP-PP}}{\partial x_0} = \frac{m (2 - x_0 - x_1)}{4} > 0,
\]
taking only the demand expansion effect into account thus indicates that \( x_0^{PP-UP} > x_0^{PP-PP} = \frac{1}{3} x_1 \).

However, from equation (31), \( \frac{dp_1}{dx_0} = -\frac{1}{2} m \), hence one drawback of moving closer to firm 1 is that firm 1 will respond by setting a lower uniform price. Inserting for (31) into (29) we find that the strategic effect is equal to
\[
\left( \frac{\partial \pi_0}{\partial p_1} \frac{dp_1}{dx_0} \right)^{PP-UP} = -(2 + x_0 + x_1)m < 0,
\]
\(^{19}\)Due to symmetry \((x_0 = 1 - x_1)\) the equilibrium location in this case would be \( x_0 = \frac{1}{4} \) and \( x_1 = \frac{3}{4} \). See also Bhaskar and To (2004).
encourages firm 0 to locate further away from the rival. Adding the demand expansion effect and the strategic effect yields

\[ \frac{d\pi_{0}^{PP-UP}}{dx_0} = \frac{m(2 - 15x_0 + x_1)}{8}. \]

The first-order condition then implies that \( x_0^{PP-UP} = \frac{1}{15}x_1 + \frac{2}{15}. \) Since \( x_0^{PP-UP} - x_0^{PP-UP} = -\frac{2(2x_1 - 1)}{15} < 0, \) firm 0 will locate further away from firm 1 if firm 1 uses uniform pricing than if firm 1 uses personalized pricing. As an example, suppose that \( x_1 = 0.75. \) Then we would have \( x_0 = 0.25 \) if firm 1 use personalized pricing, while we would have \( x_0 \approx 0.18 \) if firm 1 uses uniform pricing.

One implication of personalized pricing by the rival on a firm’s location incentives is therefore that the firm does not need to consider any strategic response from the rival following the firm’s choice of location; only the market expansion effect on profit remains. In contrast, if the rival uses uniform pricing, the strategic effect induces the firm to differentiate more away from the rival in order to soften price competition. Hence, even though the firm considers a rival which uses uniform pricing as relatively soft compared to a rival which uses personalized pricing, it will nonetheless locate closer to a rival using personalized pricing since the rival will not respond by lowering prices. Since firm 0 by assumption uses personalized pricing, the result is purely driven by firm 1’s choice of price policy. Consequently, if both firms use personalized pricing, they will locate relatively close to each other. This resembles Hurter and Lederer (1985), Lederer and hurter (1986) and Bhaskar and To (2004), who find that firms locate so as to minimize social costs. However, since they assume both firms use personalized pricing, they do not identify that the effect stems from the rival using personalized pricing, not firms using personalized pricing.

From Proposition 1, we then reach the following:

**Corollary 1:** Suppose firms are symmetric \( (m_0 = m_1 = m) \). Then,

(a) a firm will locate closer to a rival which uses personalized pricing compared to a rival which uses uniform pricing.

(b) if both firms use personalized pricing, they have incentives to locate relatively close to each other.
4.3 Multihoming consumers

Traditionally, consumers are restricted to buy at most one of the two goods that are offered in standard Hotelling models (which means that \(D_0 + D_1 \leq 1\)). We now relax this assumption by allowing consumers to buy one unit from each firm (multi-purchasing). We follow the concept of incremental pricing by Anderson et al. (2017b). The net utility of buying only good \(i\) is still given by equation (1), \(u_i(x) = v - m_i |x - x_i| - p_i(x)\), while the value of buying good \(i\) in addition to good \(j\) (its incremental value) equals

\[
u_{ji} = \theta [v - m_i |x - x_i|] - p_i(x),
\]

where the parameter \(\theta \in [0, 1]\). If \(\theta < 1\), the incremental value of each good is smaller than its stand-alone value, for instance due to overlap in the goods’ area of use.\(^{20}\)

Let \(x_{10}\) denote the consumer who is indifferent between buying only good 1 and buying both goods. The location of this consumer is found by solving \(u_1 = u_1 + u_{10}\). This yields

\[
x_{10} = \frac{\theta v - p_0(x)}{\theta m_0}.
\]

Note that \(x_{10}\) depends only on firm 0’s price and mismatch cost, not on the rival’s price and mismatch cost: The attractiveness of buying good 0 in addition to good 1 only hinges on the net utility offered by good 0.

The location of the consumer who is indifferent between buying only good 0 and buying both goods is likewise given by

\[
x_{01} = 1 - \frac{\theta v - p_1(x)}{\theta m_1}.
\]

We will analyze a market structure with partial multihoming. This means that some consumers buy both goods (\(D_0 + D_1 > 1\)), but none of the goods are sold to all consumers (\(D_i < 1\)). This market outcome is illustrated in Figure 2.\(^{21}\) Demand for firm \(i\)’s good and

\(^{20}\)Foros, Kind and Wyndham (2018) provide an alternative utility formulation that illustrates that the outcome does not depend on consumers having a first and a second choice. However, their analysis does not consider personalized pricing and endogenous mismatch costs.

\(^{21}\)Since the line has length 1, consumers located at \(x < 1/2\) are closer to firm 1 and therefore have good 1 as their most preferable good. Likewise, consumers located at \(x > 1/2\) are closer to firm 2 and have good 2 as their most preferable good. Hence, it follows that \(\hat{x} = 1/2\). This implies that multihoming consumers to the left of \(\hat{x}\) buy good 2 for its incremental value over good 1, while multihoming consumers to the right of \(\hat{x}\) buy good 1 for its incremental value over good 2.
the distribution of singlehoming (SHC) and multihoming (MHC) consumers are (where $x_i$ is firm $i$’s location)

$$D_i = \left| x_{ij} - x_i \right|_{\text{SHC}} + \left| x_{ji} - x_{ij} \right|_{\text{MHC}} = \left| x_{ji} - x_i \right|.$$  \hspace{1cm} (35)

Hence, total demand for good 0 is $D_0 = x_{10}$, total demand for good 1 is $D_1 = 1 - x_{01}$, and the number of multihomers is given by $(x_{10} - x_{01})$.

Let us first consider the outcome when firm 0 uses uniform pricing.\(^{22}\) Its profit level is then given by $\pi_0 = (p_0 - c)D_0$. Since $D_0 = x_{10}$ is independent of $p_1$ and $m_1$, the profit maximizing price and profitability of good 0 are independent of whether firm 1 uses uniform or personalized pricing:

$$p_{0 UP-R}^* = \frac{c + v\theta}{2}, \hspace{1cm} \pi_{0 UP-R}^* = \frac{(v\theta - c)^2}{4\theta m_0}. \hspace{1cm} (36)$$

Inserting (36) into (33) we find that demand equals

$$D_{0 UP-R}^* = \frac{v\theta - c}{2\theta m_0}. \hspace{1cm} (38)$$

From (37) we note that firm 0 chooses to minimize own mismatch costs whatever the price policy of the rival.

Let us now assume that firm 0 uses personalized pricing. For reasons that become clear below, we assume that personalized pricing involves an extra marginal cost equal to $\phi > 0$. In equilibrium firm 0 then charges $p_{0 PP-R}^*(x) = v - m_0x$ towards its exclusive (singlehoming) consumers, $p_{0 PP-R}^*(x) = \theta (v - m_0x)$ towards multihoming consumers, and $p_{0 PP-R}^*(x) = c + \phi$ towards its marginal consumer (and those served by the rival). Thus, the smaller the mismatch costs are, the higher price can firm 0 charge each of its consumers.

Inserting that $p_{0 PP-R}^*(\bar{x}) = c + \phi$ into equation (33) yields

$$D_{0 PP-R}^* = \frac{\theta v - (c + \phi)}{\theta m_0},$$

\(^{22}\)It is beyond the scope of the present paper to provide a complete analysis of possible singlehoming and multihoming equilibria and their stability; we limit our attention to consider candidate equilibria with partial multihoming. See the appendix in Anderson et al. (2017b) for a comprehensive analysis of deviation incentives.
which shows that firm 0’s total sales are decreasing in \( m_0 \). By reducing mismatch costs, the firm will therefore both be able to charge a higher price and sell more since the number of exclusive consumers for firm 0 is independent of \( m_0 \), cf. equation (34). Hence, also in this case, the firm minimizes its own mismatch costs independently of which price policy the rival uses. If firm 1 also uses personalized pricing, firm 0’s equilibrium profit is

\[
\pi_0^{PP-PP} = \int_0^{x_{01}} (v - mx - c - \phi) \, dx + \int_{x_{01}}^{x_{10}} (\theta (v - mx) - c - \phi) \, dx
\]

\[
= \frac{2(v - c - \phi) - mx_{01}}{2} x_{01} + \frac{2(v\theta - c - \phi) - \theta m(x_{01} + x_{10})}{2} (x_{10} - x_{01}),
\]

where \( x_{10} = \frac{\theta v - (c + \phi)}{\theta m} \) and \( x_{01} = 1 - \frac{\theta v - (c + \phi)}{\theta m} \).

From the above discussion, if consumers multihome, firms cannot affect the rival’s price policy through its choice of mismatch costs. We can state:

**Proposition 5:** Each firm will minimize mismatch costs, independently of which price policy the rival uses, if some consumers multihome.

As noted above, \( x_{10} \) only depends on firm 0’s price and mismatch cost, thus firm 0’s total demand is independent of the rival’s actions. On the other hand, since \( x_{01} \) only depends on firm 1’s price and mismatch cost, firm 1 can by its actions affect firm 0’s demand composition. Specifically, a reduction in \( m_1 \) expands firm 1’s demand by turning some of firm 0’s exclusive consumers into multihomers. If firm 0 uses uniform pricing, the demand composition does not matter for its profit since singlehomers and multihomers are charged the same price. However, if firm 0 uses personalized pricing, a reduction in \( m_1 \) hurts firm 0 because a multihomer is only worth \( \theta \) of a singlehomer. Further, from Proposition 5, we know that firms are incentivized to minimize their mismatch costs independently of what the rival does. We then reach the following:

**Corollary 2:** Assume some, but not all, consumers are multihoming. If firm \( i \) uses uniform pricing, it is not affected by the rival’s choice of uniform pricing or personalized pricing. In contrast, if firm \( i \) uses personalized pricing, it is better off if the rival uses uniform pricing.

Note that the ratio of total demand under uniform pricing and personalized pricing is

\[
\frac{D_{PP-PP}^{UP-PP}}{D_{UP-UP}^{UP-UP}} = 2(1 - \frac{\phi}{v\theta - c}).
\]

27
If $\phi = 0$, the demand is twice a large under personalized pricing than under uniform pricing, which means that the market is not covered under uniform pricing.\textsuperscript{23} Therefore, we assume an extra marginal cost $\phi > 0$ under personalized pricing to avoid this issue.

5 Concluding remarks

In a duopoly model, we examine how a firm’s incentives to reduce its mismatch cost depends on its own and on its rival’s choice between uniform pricing and personalized pricing. While a rival which uses personalized pricing will not strategically respond to a firm’s decisions on its mismatch cost, a rival using uniform pricing will respond aggressively by reducing its price if the firm lowers its mismatch cost. Therefore, firms’ incentives to change their mismatch cost depend only on the rival’s choice between uniform and personalized pricing. Firms might commit to uniform pricing in order to avoid an aggressive response from the rival in lowering its mismatch cost, which is detrimental for the firm’s profit since it loses market shares.

We let firms endogenously decide whether to commit to uniform pricing as well as the level of the non-price variable prior to the price competition stage. These assumptions allow us to examine the relationship between price policy commitments by either firm and strategic commitments in the non-price variable. As non-price variables we consider the mismatch cost in our main model and location incentives in an extension.

Therefore, we also contribute to the literature on personalized pricing by examining how non-price commitments in general depend on the commitment to a uniform price policy. It has been pointed out in previous works that a firm which uses personalized pricing optimally sets price equal to marginal cost in the rival’s market region (Lederer and Hurter, 1986; Thisse and Vives, 1988). Given that the choice of the non-price variable is observable prior to the price competition stage, this means that the strategic effect of a firm’s choice of non-price commitment in stage 2 ceases to exist if it faces a rival which uses personalized pricing. We show that it is not price discrimination in itself that removes strategic effects of non-price commitments, it is price discrimination by the rival, and the rival only, that drives the result. The choice of whether to commit to uniform pricing in stage 1 can therefore be seen as a choice of whether to give the rival strategic incentives

\textsuperscript{23} Partial multihoming implies that the total demand is strictly less than 2.
to undertake non-price commitments in stage 2. To our knowledge, this has not yet been highlighted in the previous literature.

Our analysis highlights one potential force which may incentivize firms to continue using uniform pricing as the pricing standard even when they are capable of practicing personalized pricing. Due to rapid developments in machine learning and data collection technologies, which improve firms’ capability of practicing personalized pricing as well as offering tailored products, both personalized pricing and product tailoring have been devoted great attention recently from the media (e.g. Forbes, 2014) as well as from the academic literature (e.g. Esteves, 2010; Valletti and Wu, 2016; Prüfer and Schottmüller, 2017). Our results can help explain why firms are slower to adapt personalized pricing than one would expect, despite that they have the technology and information to do so.
6 References


Forbes (2014). Different Customers, Different Prices, Thanks To Big Data. April 14. Available at: https://www.forbes.com/sites/adamtanner/2014/03/26/different-customers-
different-prices-thanks-to-big-data/#24ef3add5730. [Accessed 4 April 2018].


